

八十四學年度 原科所 所 業 乙 組碩士班研究生入學考試

科目 應用數學 科號 3105 共 二 頁第 一 頁 \*請在試卷【答案卷】內作答  
3205

(16%) 1. Solve the following ordinary differential equations:

(8%) (a)  $y'' + 4y = r(t), \quad y(0)=0, y'(0)=3$   
 where  $r(t) = 3\sin t \quad 0 < t < \pi$   
 $-3\sin t \quad t > \pi$

(8%) (b)  $x^2y' = y^2 + 3xy + x^2$

(15%) 2. Find the eigenvalue and eigenfunction of the following problem:

$$y'' + \lambda y = 0, \quad y(0) = y(2\pi), \quad y'(0) = y'(2\pi)$$

verify the eigenfunction forms an orthogonal set.

(12%) 3. For the following system of equations,

$$\begin{aligned} 5x - 3y + 4z &= -23 \\ -x + 2y - 8z &= 29 \\ 4y + 7z &= -13 \end{aligned}$$

(4%) (a) Determine the rank of the coefficient matrix.

(4%) (b) Find the inverse of the coefficient matrix.

(4%) (c) Find the solution.

(16%) 4. For  $\vec{F} = x^3 \hat{i} + y^3 \hat{j} + 3z(2 - x^2 - y^2) \hat{k}$ ,  
 $S: 9x^2 + y^2 + 9z^2 = 9$

(8%) (a) Evaluate the surface integral  $\iint_S \vec{F} \cdot \hat{n} \, dA$   
 where  $\hat{n}$  is the outer unit normal vector of  $S$ .

(8%) (b) Find the surface area of  $S$ .

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- (15%) 5. The deflection  $u$  of a vibrating membrane is governed by a two-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Find the vibration of a circular membrane of radius  $R$  with

*boundary condition*  $u(R, t) = 0$

*initial condition*  $u(r, 0) = f(r); \quad \frac{\partial u}{\partial t} \Big|_{t=0} = g(r)$

- (16%) 6. Evaluate the following integrals:

(8%) (a)  $\int_0^{\infty} e^{-x^2} dx$

(8%) (b)  $\int_0^{\infty} e^{-x^2} \cos 2bx dx$

- (10%) 7. Find the mean and variance of  $Y = -4X + 5$ , where  $X$  is the random variable which has the density

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$