

國立清華大學 104 學年度碩士班考試入學試題

系所班組別：生醫工程與環境科學系 丙組 (放射組)

考試科目 (代碼)：工程數學(2401)

共__1__頁，第__1__頁 *請在【答案卷、卡】作答

1. Solve the given initial-value problem. (10 pts.)

$$(1+x^2) dy/dx + 2xy = f(x), y(0) = 0, \text{ where } f(x) = \begin{cases} x, & 0 \leq x < 1 \\ -x, & x \geq 1 \end{cases}$$

2. Solve the given differential equation

$$(a) (10 - 6y + e^{-3x}) dx - 2 dy = 0 \quad (10 \text{ pts.})$$

$$(b) y'' + 4y' + 5y = 35e^{-4x}, \quad y(0) = -3, y'(0) = 1 \quad (10 \text{ pts.})$$

3. Use Laplace Transform to solve $y(t)$ (10 pts.)

$$y'' + 4y = \sin t \mathcal{U}(t-2\pi), y(0) = 1, y'(0) = -1$$

4. Solve the given differential equation by variation of parameters

$$x^2 y'' - 3x y' + 3y = 2x^4 e^x \quad (10 \text{ pts.})$$

5. Find the eigenvalues and eigenvectors of the given matrix.

$$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix} \quad (10 \text{ pts.})$$

6. Show that the set $\{1, \cos x, \cos 2x, \cos 3x, \dots, \cos nx, \dots\}$ is orthogonal on the interval $[-\pi, \pi]$. (10 pts.)

7. Solve $\mathbf{X}' = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} \mathbf{X}$, $\mathbf{X}(0) = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$ (10 pts.)

8. Find the complex Fourier series of the $f(x)$ on the given interval.

$$f(x) = \begin{cases} 0, & -\frac{1}{2} < x < -\frac{1}{4} \\ 1, & -\frac{1}{4} < x < \frac{1}{4} \\ 0, & \frac{1}{4} < x < \frac{1}{2} \end{cases} \quad (10 \text{ pts.})$$

9. Find the temperature $u(x,t)$ in a rod of length 10 if the initial temperature is $f(x)=100-10x$ and if the temperature at both ends are kept 50 all the time. The thermal diffusivity constant k of this rod is 4. (10 pts.)