國立清華大學 103 學年度碩士班考試入學試題

系所班組別：生醫工程與環境科學系　丙組(醫學物理與工程組)

考試科目（代碼）：工程數學(2503)

1. Calculate $y(t) = x(t) * h(t)$ (symbol * denotes convolution), where

$$x(t) = \begin{cases} 1, & 3 \le n \le 8 \\ 0, & otherwise \end{cases}$$

$$h(t) = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & otherwise \end{cases}$$ (10 pts)

2. (a) Find the Fourier transform of $x(t)$.

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$$

(b) Use (a) result to calculate $\int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega$

(*Hint*: Parseval's Relation: $\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$)　(10 pts)

3. Use Laplace Transform to solve $y(t)$

$$\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = 1 - u(t-1) \qquad y(0) = 0, \ \frac{d}{dt} y(0) = 1$$

(10pts)

4. A triangular pulse signal $x(t)$ is depicted in Fig. 1.

(a) Use $u(t)$ (unit step function) to describe $x(t)$.

(b) Sketch and label: $x(3t)\,u(t)$

(c) Sketch and label: $x(-2t-1)$

(d) Sketch and label: $dx(t)/dt$

(e) Sketch and label: $x(t-1)\,[\delta(t-0.5) + \delta(t-1.5)]$

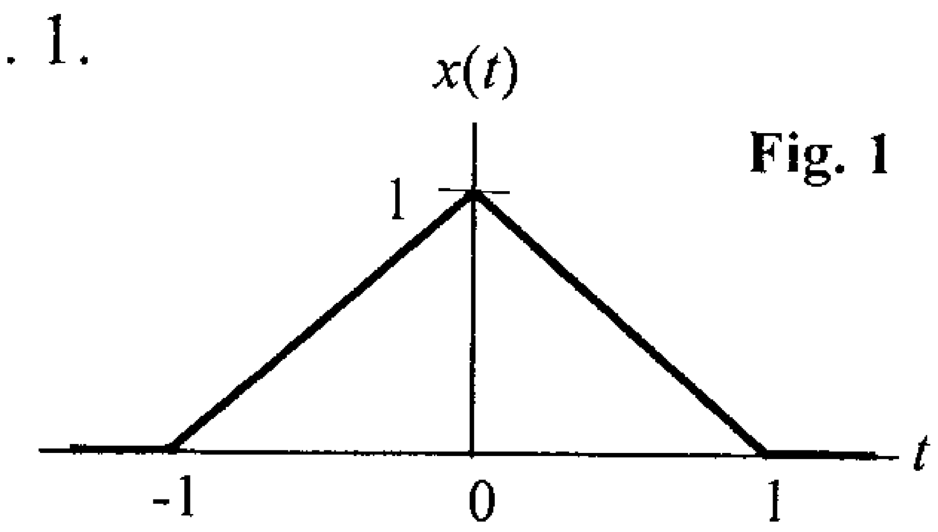

Fig. 1

(10 pts)

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5. Solve the differential equations:

(a) $x\frac{dy}{dx} - y = x^2 \sin x$ (5 pts)

(b) $\left(\frac{3y^2 - t^2}{y^5}\right)\frac{dy}{dt} + \frac{t}{2y^4} = 0, y(1) = 1$ (5 pts)

(c) $\frac{d^2x}{dt^2} + 25x = 100\sin 5t,\ x(0) = 0, x'(0) = 0$ (5 pts)

(d) $y'' + 2y(y')^3 = 0$ (5 pts)

6. Find the eigenvalues and eigenvectors of the given matrix.

$$\begin{pmatrix} 0 & 4 & 0 \\ -1 & -4 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$ (10 pts)

7. Let $\boldsymbol{A}$ be a square matrix with real entries. The $\lambda = \alpha + i\beta, \beta \neq 0,$ is a complex eigenvalue of $\boldsymbol{A}$, and $\boldsymbol{K}$ is an eigenvector corresponding to λ. Proof that the conjugate $\bar{\lambda}$ and $\overline{\boldsymbol{K}}$ is also an eigenvalue and a corresponding eigenvector. (10 pts)

8. Solve the given linear system.

$$\boldsymbol{X}' = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix}\boldsymbol{X} + \begin{pmatrix} 1 \\ \cot t \end{pmatrix}$$ (10 pts)

9. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with the given boundary conditions:

$\left.\frac{\partial u}{\partial x}\right|_{x=0} = u(0, y),\quad u(\pi, y) = 1,\ u(x, 0) = 0,\ u(x, \pi) = 0.$ (10 pts)