

國立清華大學 103 學年度碩士班考試入學試題

系所班組別：生醫工程與環境科學系 甲組(分子生醫工程組)

考試科目 (代碼)：工程數學(2303)

共 2 頁，第 1 頁\*請在【答案卷、卡】作答

1. Calculate  $y(t) = x(t) * h(t)$  (symbol \* denotes convolution), where

$$x(t) = \begin{cases} 1, & 3 \leq t \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} 1, & 4 \leq t \leq 15 \\ 0, & \text{otherwise} \end{cases} \quad (10 \text{ pts})$$

2. (a) Find the Fourier transform of  $x(t)$ .

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$$

(b) Use (a) result to calculate  $\int_{-\infty}^{\infty} \frac{\sin^2 \omega}{\omega^2} d\omega$

$$(\text{Hint: Parseval's Relation: } \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega) \quad (10 \text{ pts})$$

3. Use Laplace Transform to solve  $y(t)$

$$\frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2y(t) = 1 - u(t-1) \quad y(0) = 0, \quad \frac{d}{dt} y(0) = 1 \quad (10 \text{ pts})$$

4. A triangular pulse signal  $x(t)$  is depicted in Fig. 1.

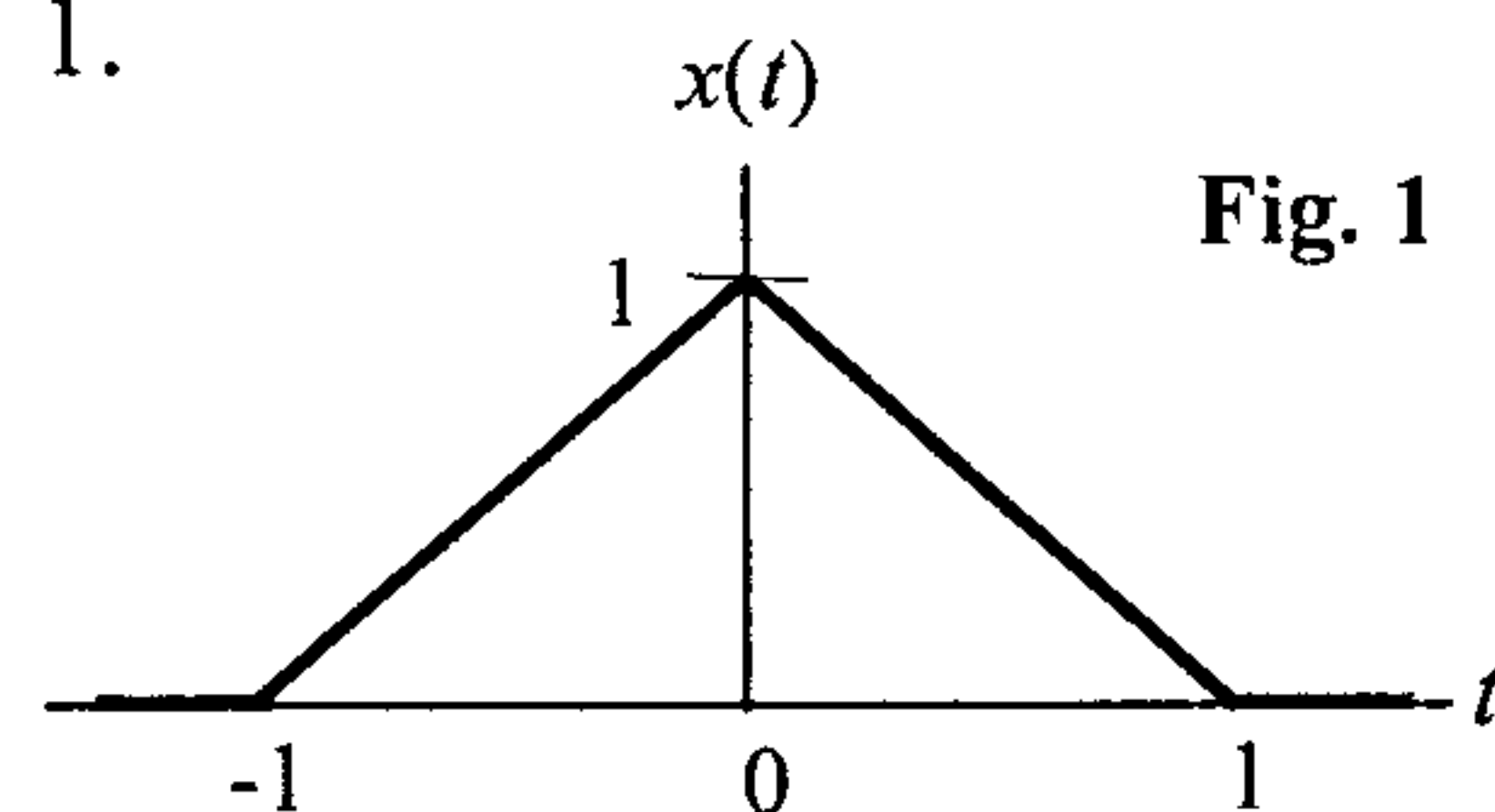
(a) Use  $u(t)$  (unit step function) to describe  $x(t)$ .

(b) Sketch and label:  $x(3t) u(t)$

(c) Sketch and label:  $x(-2t-1)$

(d) Sketch and label:  $\frac{dx(t)}{dt}$

(e) Sketch and label:  $x(t-1) [\delta(t-0.5) + \delta(t-1.5)]$



(10 pts)

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5. Solve the differential equations:

(a)  $x \frac{dy}{dx} - y = x^2 \sin x$  (5 pts)

(b)  $\left(\frac{3y^2-t^2}{y^5}\right) \frac{dy}{dt} + \frac{t}{2y^4} = 0, y(1) = 1$  (5 pts)

(c)  $\frac{d^2x}{dt^2} + 25x = 100 \sin 5t, x(0) = 0, x'(0) = 0$  (5 pts)

(d)  $y'' + 2y(y')^3 = 0$  (5 pts)

6. Find the eigenvalues and eigenvectors of the given matrix.

$$\begin{pmatrix} 0 & 4 & 0 \\ -1 & -4 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (10 \text{ pts})$$

7. Let  $A$  be a square matrix with real entries. The  $\lambda = \alpha + i\beta, \beta \neq 0$ , is a complex eigenvalue of  $A$ , and  $K$  is an eigenvector corresponding to  $\lambda$ . Prove that the conjugate  $\bar{\lambda}$  and  $\bar{K}$  is also an eigenvalue and a corresponding eigenvector. (10 pts)

8. Solve the given linear system.

$$X' = \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} X + \begin{pmatrix} 1 \\ \cot t \end{pmatrix} \quad (10 \text{ pts})$$

9. Solve the Laplace's equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  with the given boundary conditions:

$$\frac{\partial u}{\partial x} \Big|_{x=0} = u(0, y), \quad u(\pi, y) = 1, \quad u(x, 0) = 0, \quad u(x, \pi) = 0. \quad (10 \text{ pts})$$