

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：生醫工程與環境科學系(0525) 丙組(醫學物理與工程組)

考試科目 (代碼)：應用數學(2703)

共 2 頁，第 1 頁 *請在【答案卷、卡】作答

Solve the following ordinary differential equations (Problems 1-3).

1. $y \ln y dx + (x - \ln y) dy = 0$ (8 pts)

2. $xy' = y \ln(xy)$ (8 pts)

3. $y'' - 2y' + 5y = e^x \cos 2x$ (8 pts)

Solve the following ordinary differential equations by Laplace transform only (problems 4 and 5).

4. $y'' + y = f(t)$ $y(0) = 0$, $y'(0) = 1$ (8 pts)

$$f(t) = \begin{cases} 0 & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ 0 & t > 2\pi \end{cases}$$

5. A differential equation system

$$\frac{dx}{dt} = 4x - 2y + 2u(t-1) \quad (10 \text{ pts})$$

$$\frac{dy}{dt} = 3x - y + u(t-1)$$

$$x(0) = 0, \quad y(0) = \frac{1}{2}$$

6. Find the point on the surface $Z = x^2 + y^2$ at which the gradient is parallel to the

vector $4\mathbf{i} + \mathbf{j} + \frac{1}{2}\mathbf{k}$ (8 pts)

7. Find the eigenvalues and normalized eigenvectors of the given matrix

$$\begin{pmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad (8 \text{ pts})$$

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8. Find a 2x2 matrix that has eigenvalues 2 and 3 and the corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (8 \text{ pts})$$

9. $f(x) = \begin{cases} -\pi, & -2\pi < x < -\pi \\ x, & -\pi < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$, expand $f(x)$ in an appropriate Fourier sine or

cosine series. (8 pts)

10. Proof that the Fourier transform of a real odd function is an imaginary function.

(8 pts)

11. Solve the wave equation $4 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ of a string of length of 3 subject to the given

$$\text{condition: } u(0,t)=0, u(3,t)=0, u(x,0)=0, \frac{\partial u}{\partial t} \Big|_{t=0} = 2 \sin \frac{\pi}{3} x + \sin \pi x$$

(10 pts)

12. $\mathbf{F} = y^2 \mathbf{i} + xz^3 \mathbf{j} + (z-1)^2 \mathbf{k}$; Use the divergence theorem to find the outward flux of the region bounded by cylinder $x^2 + y^2 = 16$ and the planes $z=1, z=5$.

(8 pts)