國立清華大學命題紙

99學年度核子工程與科學研究所甲組、聯合招生(工科丙組、先進光源工科組)碩士班入學考試

科目 近代物理 科目代碼 2803、9802 共 / 頁，第 / 頁 *請在【答案卷卡】作答

Useful constants: $c = 3.00\times10^{8}$ m/s, $k_B = 1.38\times10^{-23}$ J/K, $h = 6.63\times10^{-34}$ J·s, $e = 1.60\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ Kg, $\mu_B = 9.274\times10^{-24}$ J/T, $R = 1.097\times10^{7}$ m^{-1}.

1. (15%) An unstable particle having a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly away with speeds of 0.987 c and 0.868 c, respectively. Find the rest masses of these two fragments.

2. (15%) Photons of wavelength 450 nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20 cm by a magnetic field whose strength is equal to 2.0×10^{-5} T. What is the work function of the metal?

3. (20%) Derive the Energy levels of hydrogen according to Bohr's quantum model of the atoms.

4. (20%) For a simple harmonic oscillator of spring constant K and mass m, one solution of the Schördinger equation is of the form $\psi(x) = Ae^{-ax^2}$.
   (A) Determine the energy of this state, and the normalization constant A.
   (B) Find the uncertainty in position Δx, and the uncertainty in momentum Δp.

5. (15%) An electron in a hydrogen atom is in a 3d state.
   (A) What is the minimum angle the angular momentum vector may make with the z-axis?
   (B) What is the most probable radius at which to find it? Given that $R_{32}(r) = Ar^2e^{-r/3a_0}$.
   (C) Make a plot to show the splitting of the 3d state due to the spin-orbit interaction.

6. (15%) Two noninteracting identical particles in a one-dimensional box (0, L) occupy the $n = 1$ and $n' = 2$ single-particle states.
   (A) Write down the normalized single-particle wave function $\psi_n(x)$.
   (B) Write down the symmetric and antisymmetric wave functions: $\psi_s(x_1, x_2)$ and $\psi_A(x_1, x_2)$ for the two-particle system.
   (C) Consider the two electrons in the ground state helium atom. If we neglect the interaction between the electrons, then the single-particle state can be described by the set of quantum numbers: $\{n, l, m_l, m_s\}$. Write down the system wave function of the ground state helium atom.