國 立 清 華 大 學 命 題 紙

99 學年度 聯合招生(工科丙組、先進光源工程組) 碩士班入學考試

科目 電磁學 科目代碼 9803 共 3 頁第 1 頁 *請在【答案卷卡】內作答

1. An *infinite* large slab of uniform charges of density $\rho = 1$ mC/m^3 and thickness $d = 1$ mm is placed adjacent to a grounded conducting plane, as shown in Fig. 1 (cross section view). (20 %)
   (a) Find the *electric field* everywhere.
   (b) Find the density of the surface charges induced on the planar conductor surface.

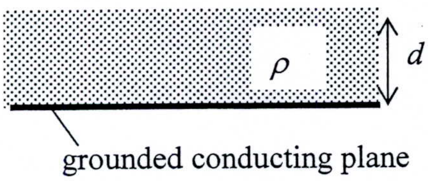

Fig. 1

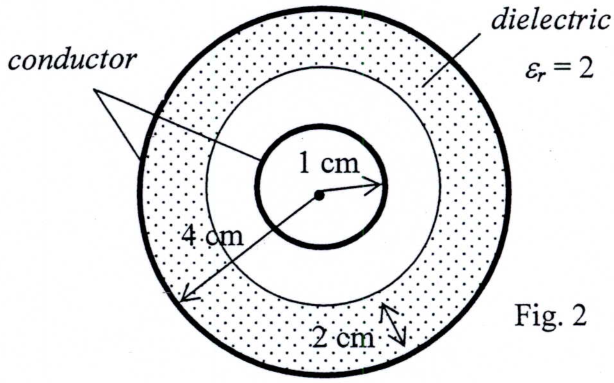

Fig. 2

2. A cylindrical coaxial cable consists of two long conducting tubes of inner and outer radii 1 cm and 4 cm, respectively. A dielectric material of dielectric constant $\varepsilon_r = 2.0$ and thickness 2 cm partially fills the space between the two conductors, as shown in Fig. 2 (cross section view). (20 %)
   (a) Find the distribution of electrostatic potential, $V(r)$, if a voltage of 10 V is applied on the inner conductor while the outer one being grounded ($V = 0$ V).
   (b) Find the *polarization* (dipole moment density) and the corresponding equivalent *bound surface charge density* at the inner surface of the dielectric (i.e., $r = 2$ cm).
   (c) Find the *capacitance* per unit length of this coaxial cable.
   (d) Find the *inductance* per unit length of this cable.

3. (a) For a monochromatic (sinusoidal) plane electromagnetic wave (in free space) propagating along the x-direction, polarized in the y-direction, and having an angular frequency ω and a wavenumber k, write down the *mathematical expression* (in time domain, spatial temporal dependence) of the electric and magnetic fields for this wave.
   (b) For a plane electromagnetic wave incident obliquely onto a planar interface formed by two infinite large media of dielectric constant 1 and 2, respectively, what would be the properties of the incident wave (e.g., incident angle, polarization, which side the incident wave coming in, etc), so that
   (i) there is no reflected wave?
   (ii) there is no transmitted wave (i.e., the wave is reflected "totally")? (20 %)

4. At a certain time (e.g., $t = 0$), an electron (e^-) is found to be located at the origin and being immersed in a region of uniform static electric and magnetic fields, $\vec{E} = \hat{x}\,1$ V/m, and $\vec{B} = \hat{z}\,1$ T, respectively, as shown in Fig. 3. (20 %)

(a) Find, at $t = 0$, the total *electromagnetic force* experienced by the electron and the *work done per unit time* on the electron by the electromagnetic force, for an electron velocity of $\vec{v}(t=0) = \hat{y}\,1$ m/sec.

(b) What would be the *cyclotron frequency* (in unit of Hertz) of the electron immersed in these uniform static electric and magnetic fields?

(c) If, instead, the electron is initially at rest ($\vec{v}(t=0) = 0$), qualitatively describe the *trajectory* of the electron (for $t > 0$) due to electromagnetic forces. Briefly explain why the electron follows such a trajectory (from the electric and magnetic force on the electron).

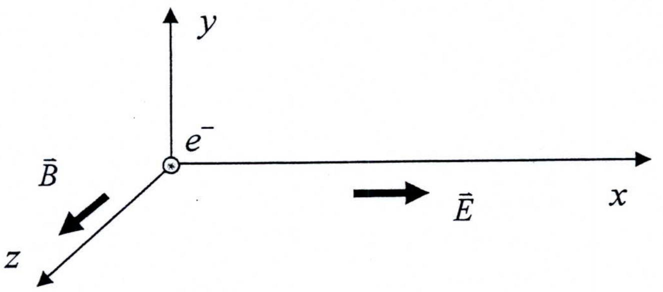

Fig. 3

5. Consider a hollow rectangular waveguide made of perfect conductor, as shown in Fig. 4. (20 %)

(a) For electromagnetic waves inside the waveguide, can the transverse electromagnetic wave (TEM) propagate along the waveguide? Explain your answer.

(b) For $a = 10$ cm, $b = 5$ cm, what is the *waveguide mode* with the *lowest* cutoff frequency? (mode type (TE or TM), index number and corresponding cutoff frequency)

(c) Continue from (b), for a wave propagating along this waveguide under the fundamental mode and at a wave frequency of twice the cutoff frequency, i.e., $f = 2 f_{cutoff}$, find the corresponding *propagation constant*, *phase* velocity and the *group* velocity.

(Equations shown in the next page may be useful but not absolutely necessary.)

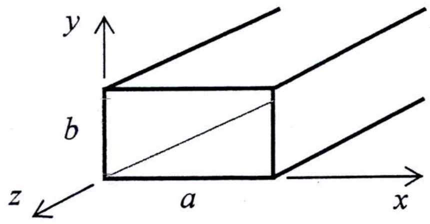

Fig. 4

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Consider a monochromatic plane wave propagating along the waveguide, in phasor (complex) expressions:

$$\left.\begin{array}{l}\text{(i)}\ \tilde{\mathbf{E}}(x,y,z,t)=\tilde{\mathbf{E}}_0(x,y)e^{i(kz-\omega t)}\\ \text{(ii)}\ \tilde{\mathbf{B}}(x,y,z,t)=\tilde{\mathbf{B}}_0(x,y)e^{i(kz-\omega t)}\end{array}\right\}\quad(1)$$

where

$$\tilde{\mathbf{E}}_0=E_x\hat{x}+E_y\hat{y}+E_z\hat{z},\quad \tilde{\mathbf{B}}_0=B_x\hat{x}+B_y\hat{y}+B_z\hat{z}\quad(2)$$

From the Maxwell's equations, one obtains

$$\left.\begin{array}{ll}\text{(i)}\ \dfrac{\partial E_y}{\partial x}-\dfrac{\partial E_x}{\partial y}=i\omega B_z, & \text{(iv)}\ \dfrac{\partial B_y}{\partial x}-\dfrac{\partial B_x}{\partial y}=-\dfrac{i\omega}{c^2}E_z\\ \text{(ii)}\ \dfrac{\partial E_z}{\partial y}-ikE_y=i\omega B_x, & \text{(v)}\ \dfrac{\partial B_z}{\partial y}-ikB_y=-\dfrac{i\omega}{c^2}E_x\\ \text{(iii)}\ ikE_x-\dfrac{\partial E_z}{\partial x}=i\omega B_y, & \text{(vi)}\ ikB_x-\dfrac{\partial B_z}{\partial x}=-\dfrac{i\omega}{c^2}E_y\end{array}\right\}\quad(3)$$

and

$$\left.\begin{array}{l}\text{(a)}\ E_x=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial E_z}{\partial x}+\omega\dfrac{\partial B_z}{\partial y}\right)\\ \text{(b)}\ E_y=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial E_z}{\partial y}-\omega\dfrac{\partial B_z}{\partial x}\right)\\ \text{(c)}\ B_x=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial B_z}{\partial x}-\dfrac{\omega}{c^2}\dfrac{\partial E_z}{\partial y}\right)\\ \text{(d)}\ B_y=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial B_z}{\partial y}+\dfrac{\omega}{c^2}\dfrac{\partial E_z}{\partial x}\right)\end{array}\right\}\quad(4)$$

Replace " i " by "$-j$" if you are more familiar with time variation in the form of " $e^{j\omega t}$ " instead of " $e^{-i\omega t}$ ".