國 立 清 華 大 學 命 題 紙

96 學年度 工程與系統科學系 （所） 乙、丙 組及先進光源學程 乙組 碩士班入學考試

科目 工程數學 科目代碼 2901、3001、3301 共 2 頁 第 1 頁 *請在【答案卷卡】內作答

1. You are required to use residues to find the value of the integral

$$\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} \qquad (-1<a<1).$$

(15%)

2. Suppose that the steady-state temperature T in a solid right circular cylinder of radius a possesses axial symmetry, and hence is of the form $T = T(r,z)$, where r is distance from the z axis. The temperature T then must satisfy the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = 0$$

inside the cylinder. Suppose that the faces $z=0$ and $z=L$ of the solid right circular cylinder are maintained at temperature zero, and that the temperature distribution along the lateral boundary $r=a$ is prescribed as $T(a,z)=f(z)$. Find the resultant steady-state temperature distribution inside the cylinder.

(15%)

3. Find the general solution of the following differential equation

$$xy' - 16 - 2y(x) - 2x^{-1} + 15x^{-2} = 0.$$

(10%)

4. Obtain, and compare the solution to

(a) $y'' + 2y' + 5y(t) = 0, \qquad y(0)=0,\ y'(0)=1;$

(b) $y'' + 2y' + 5y(t) = \delta(t), \qquad y(0)=0,\ y'(0)=0.$

where $\delta(t)$ is the Dirac delta function (unit impluse function)

$$\delta(t) = \begin{cases} \infty & \text{if } t=0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \int_0^\infty \delta(t)\,dt = 1.$$

(10%)

5. Solve the initial value problem of the first-order system

$$\begin{cases} x' = x + y \\ y' = x + y + e^{2t} \\ x(0) = y(0) = 0 \end{cases} \quad . \qquad (8\%)$$

6. Let S be the surface (with outer unit normal $\hat{n}$) of the region R bounded by the planes $z = 0,\ y = 0,\ y = 4$ and the paraboloid $z = 1 - x^2$. Compute $\iint_S \vec{F} \cdot \hat{n}\, dS$, given

$\vec{F} = (x + \sin y)\hat{i} + (2y + \cos z)\hat{j} + (3z + 4e^x)\hat{k}$.

(7%)

7. Find the surface of the torus generated by revolving the circle $(x-a)^2 + z^2 = b^2$ in xz-plane around z-axis with $b < a$.

(8%)

8. Express the periodic function $f(x) = |\cos x|$ in its Fourier series FS $f = \sum_{n=-\infty}^{\infty} c_n \exp(i2nx)$. Work out $c_n = ?$

(7%)

9. Use power series method to solve

$$y'' + 12y' + x^3 y(x) = 0.$$

Find at least five terms of the general solution.

(10%)

10. Find the inverse Laplace transform of

$$\frac{e^{-5s}}{s(s^2+12)}$$

(10%)