1. You are required to use residues to find the value of the integral

\[ \int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta} \quad (-1 < a < 1). \]

(15%) 

2. Suppose that the steady-state temperature \( T \) in a solid right circular cylinder of radius \( a \) possesses axial symmetry, and hence is of the form \( T = T(r, z) \), where \( r \) is distance from the \( z \) axis. The temperature \( T \) then must satisfy the equation

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = 0 \]

inside the cylinder. Suppose that the faces \( z = 0 \) and \( z = L \) of the solid right circular cylinder are maintained at temperature zero, and that the temperature distribution along the lateral boundary \( r = a \) is prescribed as \( T(a, z) = f(z) \). Find the resultant steady-state temperature distribution inside the cylinder.

(15%) 

3. Find the general solution of the following differential equation

\[ xy' - 16 - 2y(x) - 2x^{-1} + 15x^{-2} = 0. \]

(10%) 

4. Obtain, and compare the solution to

(a) \( y'' + 2y' + 5y(t) = 0, \quad y(0) = 0, \quad y'(0) = 1; \)

(b) \( y'' + 2y' + 5y(t) = \delta(t), \quad y(0) = 0, \quad y'(0) = 0. \)

where \( \delta(t) \) is the Dirac delta function (unit impulse function)

\[ \delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \int_0^\infty \delta(t) dt = 1. \]

(10%)
5. Solve the initial value problem of the first-order system
\[
\begin{aligned}
x' &= x + y \\
y' &= x + y + e^{2t} \\
x(0) &= y(0) = 0
\end{aligned}
\]  

(8%)

6. Let \( S \) be the surface (with outer unit normal \( \hat{n} \)) of the region \( R \) bounded by the planes \( z = 0, y = 0, y = 4 \) and the paraboloid \( z = 1 - x^2 \). Compute \( \iint_S \mathbf{F} \cdot \hat{n} \, dS \), given
\[
\mathbf{F} = (x + \sin y) \hat{i} + (2y + \cos z) \hat{j} + (3z + 4e^t) \hat{k}.
\]  

(7%)

7. Find the surface of the torus generated by revolving the circle \( (x - a)^2 + z^2 = b^2 \) in \( xz \)-plane around \( z \)-axis with \( b < a \).

(8%)

8. Express the periodic function \( f(x) = |\cos x| \) in its Fourier series \( F = \sum_{n=-\infty}^{\infty} c_n \exp(i2\pi nx) \). Work out \( c_n = ? \)

(7%)

9. Use power series method to solve
\[
y'' + 12y' + x^3 y(x) = 0.
\]
Find at least five terms of the general solution.

(10%)

10. Find the inverse Laplace transform of
\[
\frac{e^{-3s}}{s(s^2 + 12)}
\]

(10%)