94學年度 工程與系統科學 系（所）乙、丙、戊 組碩士班入學考試

科目 工程數學 科目代碼 3301, 3402, 3501 共 1 頁第 頁 *請在試卷【答案卷】內作答

1. Find the general solution of the inhomogeneous ordinary differential equation

$$y''(x)+y(x)=x\cos x \qquad (15\%)$$

2. Evaluate the determinant, and find the inverse matrix of matrix A

$$A=\begin{pmatrix} 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & -1 \end{pmatrix} \qquad (10\%)$$

3. If the Laplace transform of function f(t) is F(s), i.e. F(s) = L{f(t)},
   (a) Show that

$$L\{\int_t^\infty f(u)\,du\}=\frac{1}{s}\int_0^\infty (1-e^{-st})f(t)\,dt \qquad (6\%)$$

   (b) Evaluate

$$L\{\int_t^\infty \frac{e^{-u}}{u}\,du\} \qquad (9\%)$$

4. Find a unit vector norm to the surface S given by $z=x^3y^3+y-2$ at the point (0,0,2). (10%)

5. Let $f(x,y,z)=x^2e^{-yz}$. Compute the rate of change of f in the direction **v**= (1,1,1) at point (1,0,0). (10%)

6. Evaluate the integral $\int_S F \bullet dS$ where vector F=x **i**+y**j**+3**k** and where S is the surface of the unit sphere $x^2+y^2+z^2=1$. (10%)

7. Evaluate the integral

$$\int_0^\infty \frac{\cos ax}{1-x^4}\,dx \qquad (a>0). \qquad (15\%)$$

8. Find the time-dependent temperature distribution $u(r,\theta,t)$ in a semicircular plate $0\le r\le 1$, $0\le\theta\le\pi$, given that the straight edge of the plate formed by $0\le r\le 1$, $\theta=0$ and $\theta=\pi$ is insulated, the semicircular boundary is maintained at zero temperature, and the initial temperature distribution is $u(r,\theta,0)=(1-r)\cos\theta$. (15%)

[polar coordinates (r,θ) $\nabla^2 T=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}$].