國立清華大學命題紙

九十二學年度 工程與系統科學系（所） 丁、戊 組碩士班研究生招生考試

科目 電磁學 科號 3904 4003 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

***** Show your derivations in details !!***
Make clear all your assumptions/approximations!!

1. A parallel plate capacitor consists of two grounded conducting plates and is filled with charges of density $-\rho_0$ and $-\rho_0$ in each half space, where ρ_0=1 C/cm^3 and d = 10 mm, as shown in Fig. 1. Find the electric field $\mathbf{E}(x)$, $0 < x < d$. (15 %)

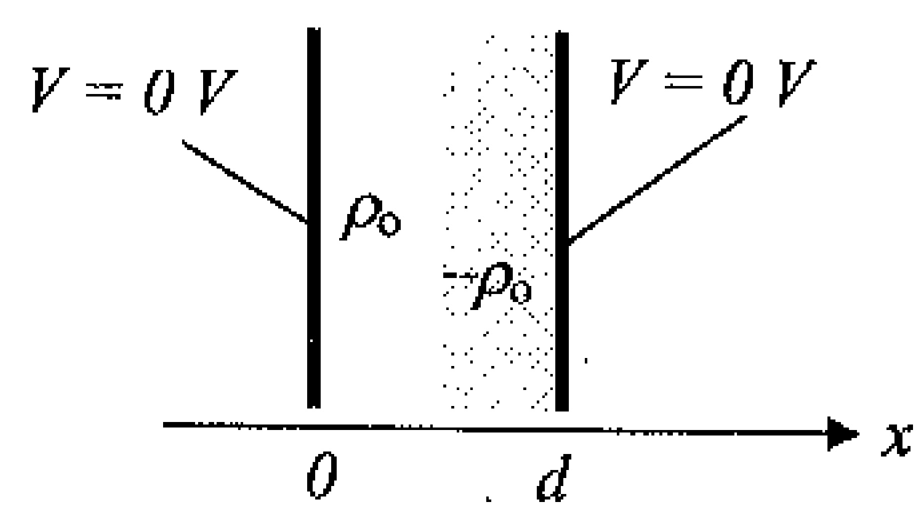


Fig. 1

2. An electric dipole **p** is placed in front of an infinite and uniform surface charge of density σ (fixed in space at $x = 0$), as shown in Fig. 2 (a). (15 %)
   (a) Find the force and torque on the dipole (in terms of a, σ and **p**, etc).
   (b) Find the force and torque on the dipole if the charge sheet is replaced by an infinite conducting surface and **p** is parallel to the surface (Fig. 2(b)).

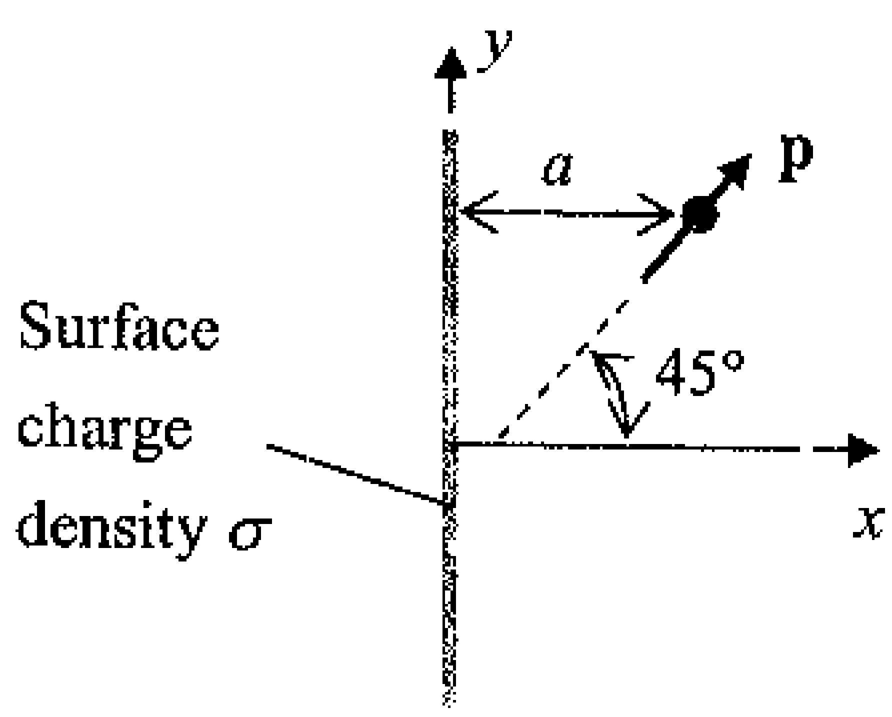


Fig. 2 (a)

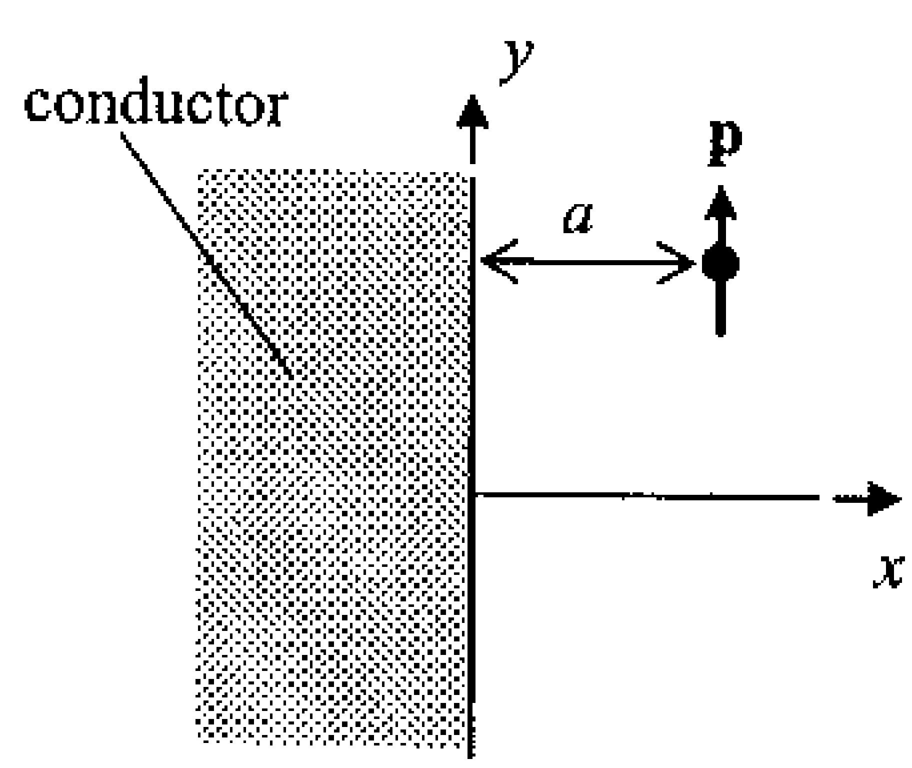
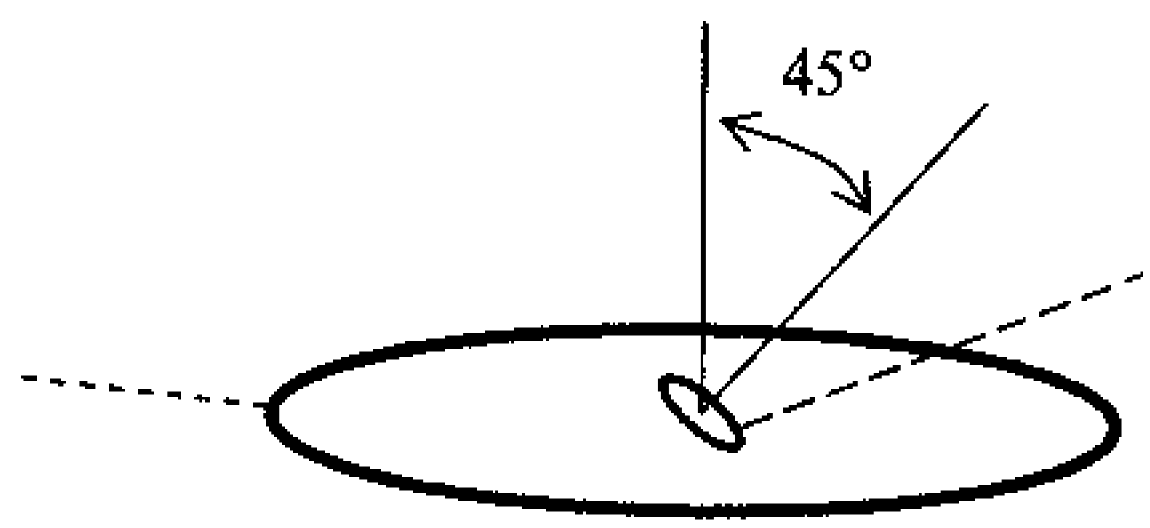


Fig. 2 (b)

3. (a) What's the physical meaning (and definition) of *mutual inductance*?
   (b) For the two circular current loops shown in Fig. 3, what is the magnetic flux generated by current in loop 1 through loop 2. Note that the two loops are placed concentric but oriented with an angle of 45° between their surface normals. (20 %)

loop 2:
radius : 10 cm
10 turns



loop 1:
radius : 1 mm
1 turns, current: 1A

Fig. 3

4. A capacitor formed by two parallel conductor of area 1 cm^2 is partially filled with a dielectric of dielectric constant $\varepsilon_r = 4$, as shown in Fig. 4. (15 %)
   (a) Find the capacitance. (neglect fringing field effect)
   (b) Find the surface charge density, $\sigma(t)$, on the top plate and voltage, $V(t)$ (with respect to the bottom plate) if a time varying (sinusoidal) current, $I(t) = I_o\cos\omega t$, where $I_o = 1$ A and frequency is 1 MHz, is applied on the top plate and the bottom plate is grounded.
   (c) Find the electric fields (including direction) between the conducting plates, $\mathbf{E}(t)$.
   (note: make reasonable assumptions if necessary)

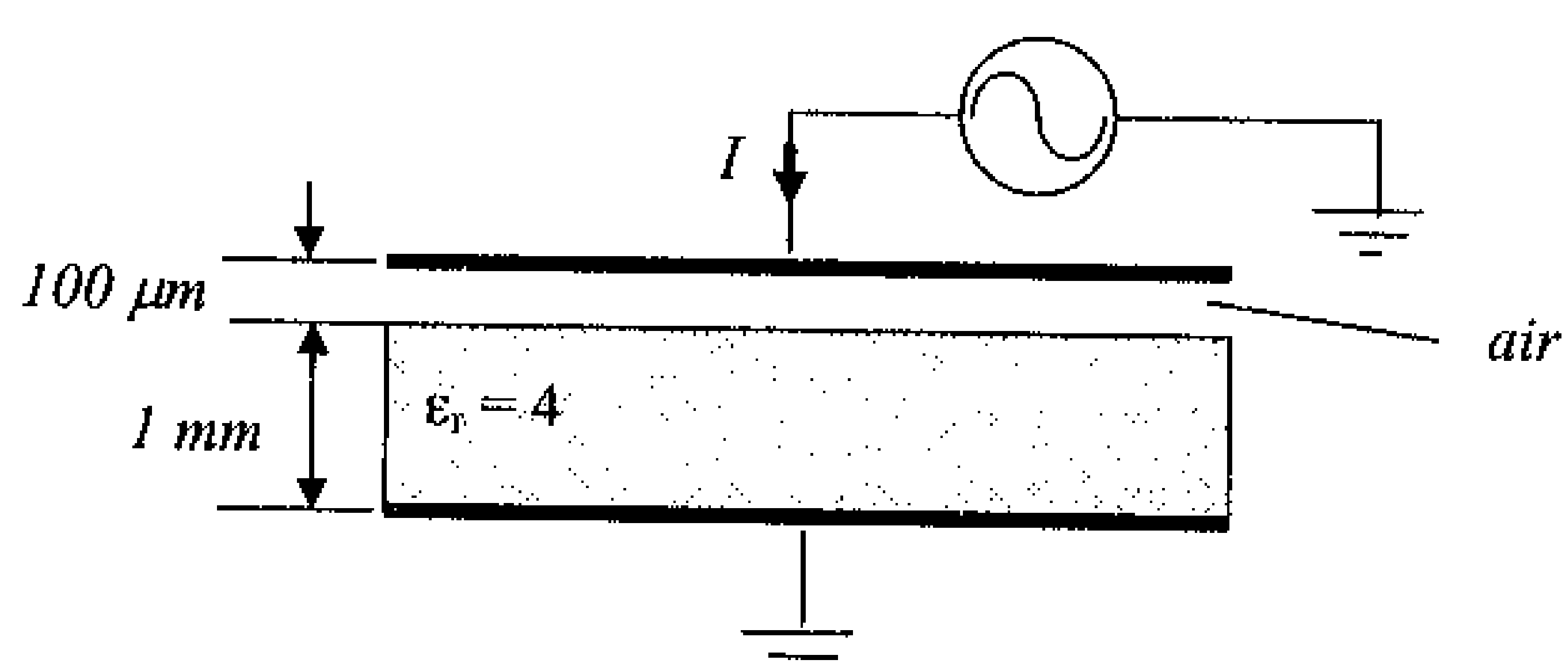
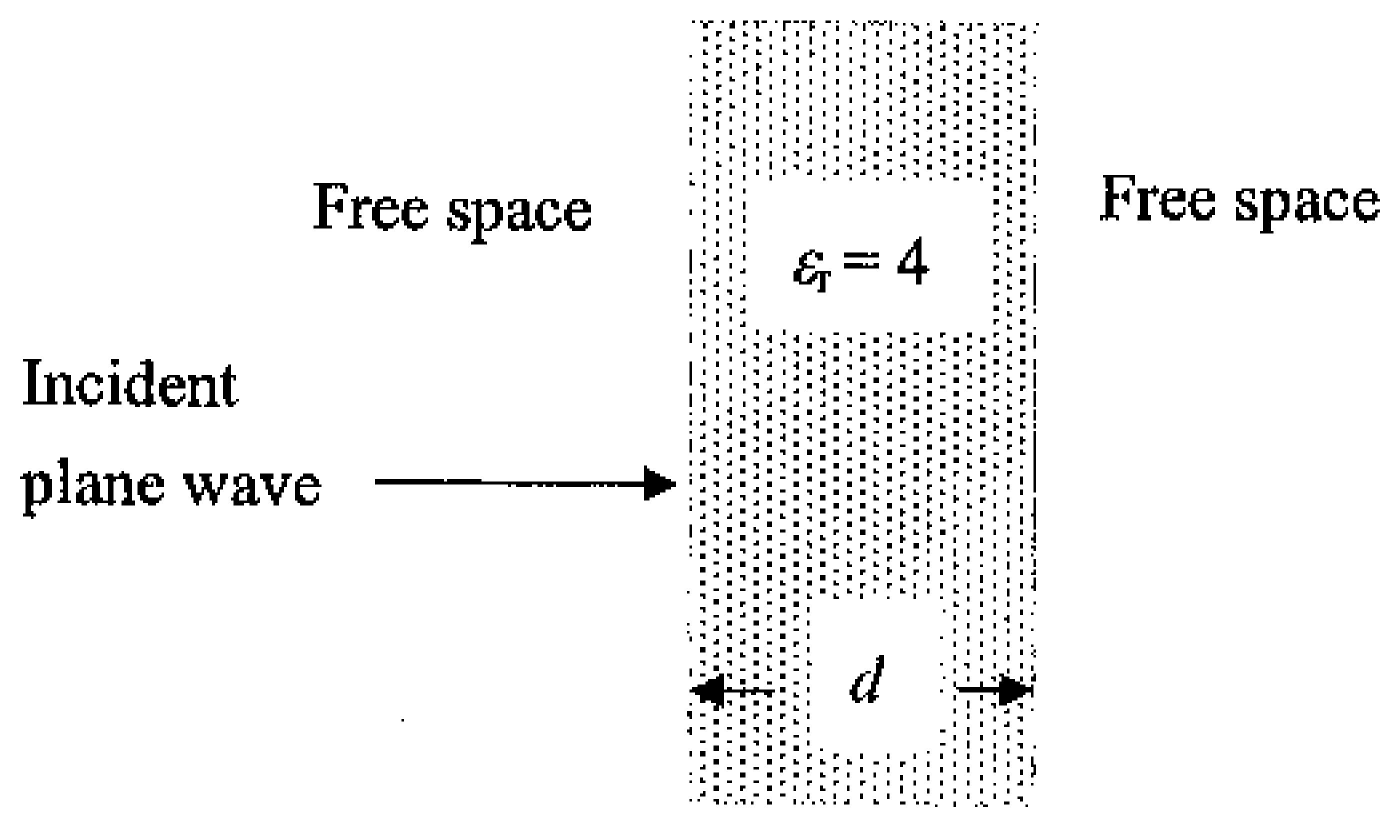


Fig. 4

5. Consider a plane electromagnetic wave incident normally on an infinite dielectric slab of dielectric constant $\varepsilon_r = 4$ and thickness d, as shown in Fig. 5.
   (a) Find the transmission coefficient, T (for electric fields) in terms of the frequency and d.
   (b) What's the minimum thickness of the slab that $T = 1$ if the wave frequency is 1 GHz. (20 %)



6. Consider a hollow rectangular waveguide made of perfect conductor, as shown in the Fig. 6, (15 %)

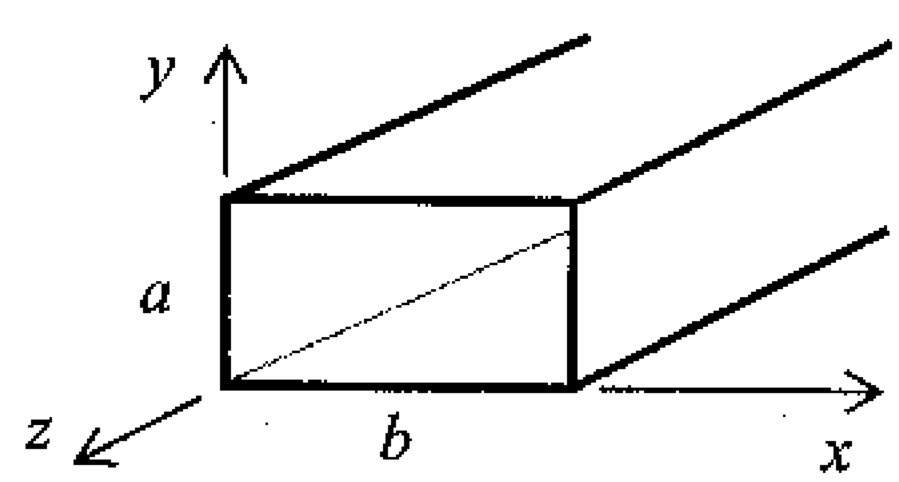


Fig. 6

(a) For electromagnetic waves inside the waveguide, can transverse electromagnetic wave (TEM) propagate along the waveguide? Explain your answer.
(b) For $a = 5$ mm, $b = 10$ mm, what is the waveguide mode with the lowest cutoff frequency?(mode type, index number and cutoff frequency) Qualitatively plot its *transverse* fields distribution pattern (electric and magnetic field lines, on x-y plane).

(Equations shown in the next page may be useful but not absolutely necessary.)

Consider a monochromatic plane wave propagating along the waveguide, in phasor (complex) expressions:

$$\left.\begin{array}{l}\text{(i)}\ \ \widetilde{\mathbf{E}}(x,y,z,t)=\widetilde{\mathbf{E}}_0(x,y)e^{i(kz-\omega t)} \\ \text{(ii)}\ \ \widetilde{\mathbf{B}}(x,y,z,t)=\widetilde{\mathbf{B}}_0(x,y)e^{i(kz-\omega t)}\end{array}\right\} \quad (1)$$

where

$$\widetilde{\mathbf{E}}_0=E_x\hat{x}+E_y\hat{y}+E_z\hat{z},\quad \widetilde{\mathbf{B}}_0=B_x\hat{x}+B_y\hat{y}+B_z\hat{z} \quad (2)$$

From the Maxwell's equations, one obtains

$$\left.\begin{array}{ll}\text{(i)}\ \ \dfrac{\partial E_y}{\partial x}-\dfrac{\partial E_x}{\partial y}=i\omega B_z, & \text{(iv)}\ \ \dfrac{\partial B_y}{\partial x}-\dfrac{\partial B_x}{\partial y}=-\dfrac{i\omega}{c^2}E_z \\ \text{(ii)}\ \ \dfrac{\partial E_z}{\partial y}-ikE_y=i\omega B_x, & \text{(v)}\ \ \dfrac{\partial B_z}{\partial y}-ikB_y=-\dfrac{i\omega}{c^2}E_x \\ \text{(iii)}\ \ ikE_x-\dfrac{\partial E_z}{\partial x}=i\omega B_y, & \text{(vi)}\ \ ikB_x-\dfrac{\partial B_z}{\partial x}=-\dfrac{i\omega}{c^2}E_y\end{array}\right\} \quad (3)$$

and

$$\left.\begin{array}{l}\text{(a)}\ \ E_x=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial E_z}{\partial x}+\omega\dfrac{\partial B_z}{\partial y}\right) \\ \text{(b)}\ \ E_y=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial E_z}{\partial y}-\omega\dfrac{\partial B_z}{\partial x}\right) \\ \text{(c)}\ \ B_x=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial B_z}{\partial x}-\dfrac{\omega}{c^2}\dfrac{\partial E_z}{\partial y}\right) \\ \text{(d)}\ \ B_y=\dfrac{i}{(\omega/c)^2-k^2}\left(k\dfrac{\partial B_z}{\partial y}+\dfrac{\omega}{c^2}\dfrac{\partial E_z}{\partial x}\right)\end{array}\right\} \quad (4)$$

Replace " i " by " $-j$ " if you are more familiar with time variation in the form of " $e^{j\omega t}$ " instead of " $e^{-i\omega t}$ ".