國 立 清 華 大 學 命 題 紙

九十二學年度 工程與系統科學 系（所） 乙、丙、丁、戊 組碩士班研究生招生考試
3701、3902、4002、

科目 工程數學 科號 3802 共 一 頁第 一 頁 *請在試卷【答案卷】內作答

1. Given that $y_1(x) = x$ is a solution of the differential equation

$$y'' - \frac{2x}{1+x^2} y' + \frac{2}{1+x^2} y = 0.$$

Find the second solution. (15%)

2. Solve the following problem:

$$4y'' - 4\,y' + 17\,y(t) = 0. \qquad y(0) = 2, \quad y'(0) = 5.$$ (10%)

3. Find the power series solutions about point $x = 0$ of the following equation:

$$x^2\,y'' + x\,y' - (x^2 + 1/4)\,y(x) = 0.$$ (15%)

4. Let the velocity of a fluid be described by $\mathbf{F} = 6xz\,\mathbf{i} + x^2y\,\mathbf{j} + yz\,\mathbf{k}$. Compute the rate at which fluid is leaving the unit cube. (15%)

5. (a) Prove that the eigenvalues of $k\mathbf{A}$, for any scalar k, are k times those of matrix $\mathbf{A}$. Are the corresponding eigenspaces the same? Explain. (7%)

(b) Evaluate $\iint_S \mathbf{n} \cdot \nabla \times \vec{F}\, dA$,

where $\vec{F} = xz\,\vec{i} - yz^4\,\vec{k}$, S: $x^2 + 4y^2 + z^2 = 4$, $x \ge 0, y \ge 0, z \ge 0$ (8%)

6. Find the steady-state temperature distribution $T(r, \theta)$ in a semicircular plate of radius 1 if

$$T(1, \theta) = u_0, \qquad 0 < \theta < \pi$$
$$T(r, 0) = 0, \qquad T(r, \pi) = u_0, \qquad 0 < r < 1$$

[in polar coordinates (r, θ), $\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}$] (15%)

7. Evaluate the integral

$$\int_0^\infty \frac{dx}{1 + x^\alpha}, \alpha > 1$$

(Hint: consider the contour shown in Fig. 1; $z = x + i\,y$) (15%)

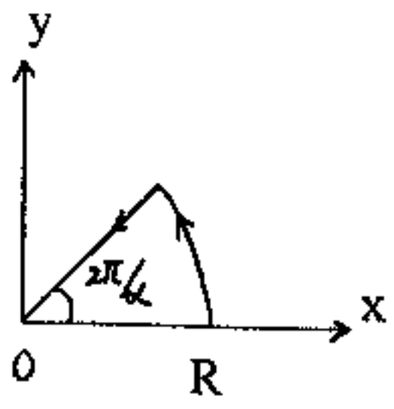

Fig. 1