

九十一學年度 工程與系統科學系(所) 丁茂 組碩士班研究生招生考試

科目 電磁學 科號 340.3 共 3 頁第 1 頁 \*請在試卷【答案卷】內作答

**\*\* Show your derivations in details !!**

**Make clear all your assumptions/approximations!!**

1. A parallel plate capacitor consists of two metal plates and is filled with charges of density  $\rho(x) = -\rho_0 x(x-d)$ , where  $\rho_0 = 10^{-6} \text{ C/m}^3$  and  $d = 2 \text{ mm}$ , as shown in Fig. 1. Find the electric field  $\mathbf{E}(x)$ ,  $0 < x < d$ . (15%)

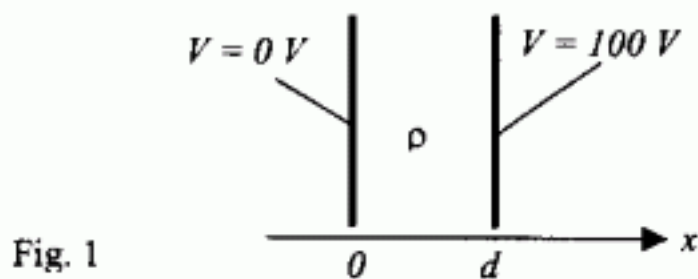


Fig. 1

2. A charge  $q$  is placed in front of a conducting corner ( $90^\circ$ ), as shown in Fig. 2. (The conductor fills all the space except for the region  $x > 0$  and  $y > 0$ .)
- Find the electric field (in terms of  $a$ ,  $x$ ,  $y$  and  $q$ ).
  - Find the charge density distribution on conductor surfaces.
  - Determine the force on  $q$ .
- (15%)

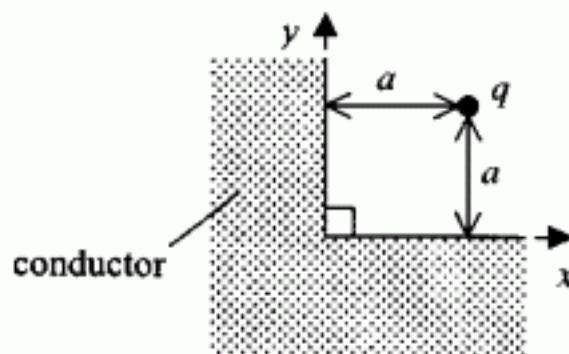


Fig. 2

3. An infinite long cylindrical solenoid of diameter  $D = 20$  cm and a winding 10 turns per cm is filled with a magnetic material tube ( $\mu_r = 1000$ ) of inner diameter  $a = 10$  cm and outer diameter 20 cm, as shown in Fig. 3.
- (a) Find the magnetic field  $\mathbf{B}$  (direction and magnitude) everywhere if the current per turn  $I = 1$  A.
- (b) Inductance per cm of the solenoid.
- (c) Magnetic energy per cm stored in the solenoid. (20%)

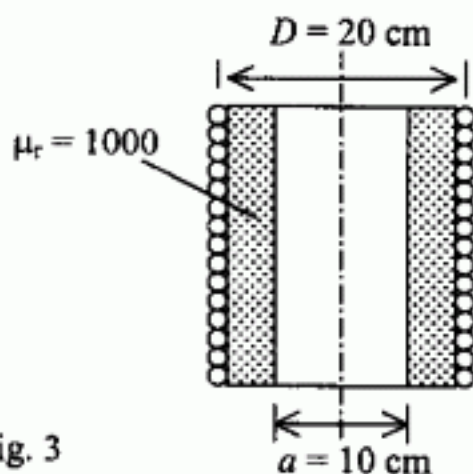


Fig. 3

4. (a) Explain the *physical meaning* of (i) phase velocity, and (ii) group velocity. (20%)
- (b) An oscillating magnetic dipole  $\mathbf{m}(t) = m_0 \cos(\omega t) \hat{\mathbf{x}}$  is located at the origin, as shown in Fig. 4. Determine the directions of the electric field, magnetic field and Poynting vector at points A, B and C, respectively. Assuming the distances from A, B and C to the origin are the same and is much larger than  $c/\omega$ , where  $c$  is the speed of light and  $\omega$  is radiation's angular frequency.
- (d) In (b), plot the "radiation pattern" (polar plot of power density distribution) on  $xy$ -plane and  $yz$ -plane.

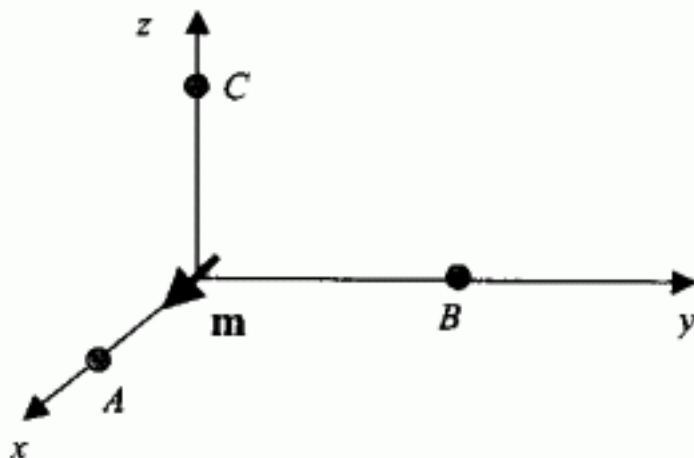


Fig. 4

5. A capacitor formed by two large parallel conductor is partly filled with two dielectrics of dielectric constant  $\epsilon_r = 10$  and  $4$ , respectively, as shown in Fig. 5.
- Find the capacitance.
  - Find the charge density on the top plate if a DC voltage of  $100\text{ V}$  is applied on the bottom plate
  - Find the *force per unit area* on the top conductor (grounded). (15 %)

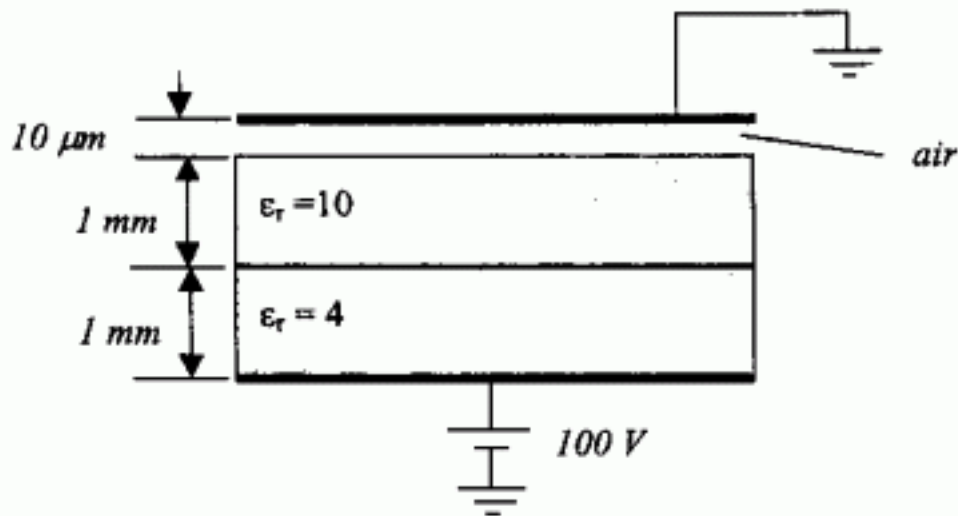


Fig. 5

6. Consider a rectangular waveguide made of perfect conductor and filled with a dielectric  $\epsilon_r = 4$ , as shown in the Fig. 6, (15 %)

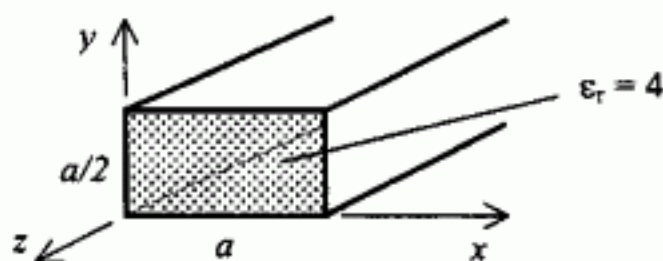


Fig. 6

- What is the waveguide mode with the lowest cutoff frequency? Qualitatively plot its dispersion diagram ( $\omega - k_z$  relation). Find the cutoff frequency if  $a = 10\text{ cm}$ .
- Find the *phase* and *group* velocities at frequency  $1\text{ GHz}$ .