

九十一學年度 工程與系統科學 系(所) 乙、丙、丁、戊 組碩士班研究生招生考試
 科目 工程數學 科號 3701, 3901, 4001 3801 共 一 頁第 一 頁 *請在試卷【答案卷】內作答

1. Find the general solution $y(x)$ of the following differential equation.

$$x^2 y'' + 3xy' + y = 0. \quad (12\%)$$

2. Find the solution $y(t)$ of the following initial value problem.

$$y'' - 3y' + 2y(t) = 20 \sin t \quad y(0) = 3, y'(0) = -6 \quad (13\%)$$

3. Find the general solution $y(x)$ of the following differential equation using power series method.

$$y'' - 4xy' + (4x^2 - 2)y = 0. \quad (15\%)$$

4. The matrix A is given as

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(a) Find the rank and null space of the matrix A . (7%)

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix A . (8%)

Total (15%)

5. Let vector $\mathbf{r}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r = (x^2 + y^2 + z^2)^{1/2}$.

(a) Find $\nabla(r^n)$ $r \neq 0$, for $n = 1, 2, 3, \dots$ (7%)

(b) Find $\nabla^2(r^n)$ $r \neq 0$, for $n = 1, 2, 3, \dots$ (8%)

Total (15%)

6. The temperatures at the ends $x = 0$ and $x = 100$ cm of a rod 100 cm in length, with insulated lateral boundaries, are held at 0°C and 100°C , respectively, until steady-state conditions prevail. Then, at the instant $t = 0$, the temperatures of the two ends are interchanged. Find the resultant temperature distribution $T(x,t)$ as a function of x and t . $T(x,t)$ satisfies the equation

$$\alpha^2 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}. \quad (15\%)$$

7. Consider a convergent Laurent series

$$f(z) = \sum_{n=-\infty}^{-1} z^n + \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}},$$

(a) find the region of convergence and the closed form of $f(z)$; (10%)

(b) find the Laurent series of $f(z)$ that is convergent in $0 < |z-1| < 1$. (5%)

Total (15%)