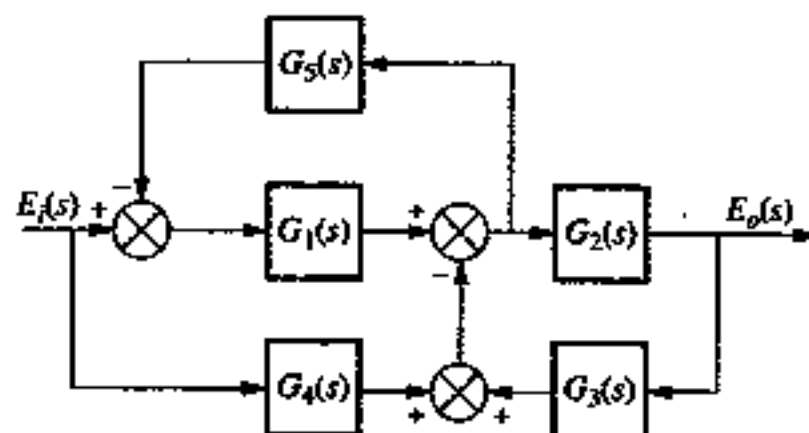


1. Reduce the system shown below to a single equivalent transfer function $T(s) = E_o(s)/E_i(s)$. (10%)



2. Consider the follow second-order system:

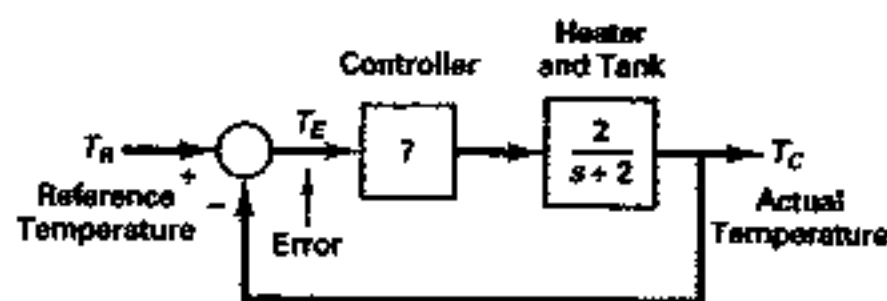
$$H(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)$$

Show that the unit step response is

$$y(t) = 1 - 1/(1 - \zeta^2)^{0.5} \exp(-\zeta\omega_n t) \sin(\omega_d t + \theta)$$

where $\omega_d = \omega_n(1 - \zeta^2)^{0.5}$ and $\theta = \cos^{-1} \zeta$ (20%)

3. Design a simplest controller for the system shown below such that the steady-state error with a step input is zero, the damping ratio is unity, and the undamped natural frequency is 4 rad/s. (10%)



4. Show that when $0 < \zeta < 0.707$, the frequency response of a system with transfer function $H(s)$, which is shown in problem 2, has a peak amplitude given by

$$1/[2\zeta(1 - \zeta^2)^{0.5}] \quad (20\%)$$

5. Consider the transfer function $G(s) = K(s+1)^2/[s(s-1)^2]$.

- If $K=1$, plot the Bode plot. (8%)
- If $K=1$, plot the polar plot. (6%)
- Plot the root locus. (6%)
- Find the range of K that will ensure stability in the unity feedback system. (3%)
- Find the value of K that will cause oscillation and the frequency of oscillation. (2%)

6. The Bode plot of the plant is shown below. The design is to met (1) phase margin $\geq 60^\circ$ and (2) gain margin $\geq 10\text{dB}$. Discuss if the phase-lead or phase-lag controller can met these conditions. You must give your reasons. (15%)

