1. Printed circuit cards are placed in a functional test after being populated with semiconductor chips. A lot contains 140 cards, and 20 are selected without replacement for functional testing. If 10 cards are defective, what is the probability that at least one defective card appears in the sample? (10%)

2. Solve the Initial Value Problem:

$$y''' - y'' - y' + y = 0$$
$$y(0) = 2, \quad y'(0) = 1, \quad y''(0) = 0$$

(10%)

3. Use power series method to solve the following problem, find at least five terms of a general solution:

$$y'' + 2xy' - y = 0$$

(10%)

4. Let $P_n(x)$ be the Legendre polynomial of order n, prove that:

$$\int_{-1}^{1} x^m P_n(x)dx = 0 \quad \text{for} \quad m = 0, 1, \ldots, n-1.$$

(10%)

5. Let s be the path $s(t)=(2t, t^2, \ln t)$, defined for $t>0$. Find the arc length of s between the points$(2,1,0)$ and $(4,4,\ln 2)$.   (10%)

6. Let vector $r(x,y,z) = (x,y,z)$ and $r =(x^2+y^2+z^2)^{1/2}$. Find

    (a) $\nabla(1/r)$, $r \neq 0$ (5%)

    (b) $\nabla^2(1/r)$, $r \neq 0$ (5%)

    (c) $\nabla \bullet (r/r^3)$ (5%)

    (d) $\nabla \times (r^r r)$.   (5%)

7. Determine the steady-state temperature at interior points of the sector $0 \leq \theta \leq \alpha$, $0 \leq r \leq a$, of a circular plate if the temperature is maintained at zero along the straight edges and at a prescribed distribution $T(a, \theta) = T_0 = $ constant when $0 < \theta < \alpha$, along the curved edge.

    [polar coordinates (r, θ)

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}].$$ (15%)

8. Use the residue theorem to find the value of the integral

$$\int_0^\infty \frac{\cos ax}{(x^2+b^2)^2}\,dx \qquad (a>0, b>0).$$ (15%)