

注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。


國立清華大學 112 學年度碩士班考試入學試題

系所班組別：工程與系統科學系
乙組

科目代碼：3101

考試科目：工程數學

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「 由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 112 學年度碩士班考試入學試題

系所班組別：工程與系統科學系碩士班 乙組(0531)

考試科目 (代碼)：工程數學 (3101)

共 2 頁，第 1 頁

*請在【答案卷】作答

1. Solve the differential equations.

(a) $\frac{dy}{dx} + 3y = 9x$, obtain $y(x)$ that subjects to $y(0) = 3$. (5%)

(b) $2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x \sec x$. (6%)

(c) $\frac{dy}{dx} = \frac{2y^2+3x}{2xy}$. (6%)

2. Find the series solution of the following differential equation about $x = 0$.

$$9x^2 \frac{d^2y}{dx^2} - 9x^2 \frac{dy}{dx} - 4y = 0. \quad (10\%)$$

You have to express the solution in the form of $y(x) = C_1y_1(x) + C_2y_2(x)$. To save time, you can only show the first four terms of $y_1(x)$ and $y_2(x)$.

3. Use the Laplace transform to solve the problem and obtain $y(t)$.

$$\frac{d^2y}{dt^2} + 4y = 4tu(t-2), \quad (8\%)$$

where $y(0) = y'(0) = 0$ and $u(t)$ is the unit step function.

4. Consider the matrix

$$M = \begin{bmatrix} 2 & 3 & 3 \\ -2 & -1 & -2 \\ 2 & 0 & 1 \end{bmatrix}.$$

(a) Find the determinant of M and obtain the inverse matrix M^{-1} . (7%)

(b) Estimate the eigenvalues and eigenvectors of M . (8%)

5. (a) Use the divergence theorem to determine the surface integral of the vector function $\mathbf{F} = xyz\mathbf{i} + (yz + 3)\mathbf{j} + (z - 2)\mathbf{k}$ on the surface S of the region $0 \leq z \leq 5$, $x^2 + y^2 \leq 9$. (5%)

(b) Use Stokes' theorem to determine the line integral of the vector function $\mathbf{F} = x^2 \ln x \mathbf{i} + y \sin(y)\mathbf{j} + \tan(z^2) \mathbf{k}$ along a close curve $C: \cos \theta \mathbf{i} + \sin \theta \mathbf{j} + \cos(2\theta + 1) \mathbf{k}$, $0 \leq \theta \leq 2\pi$. (5%)

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考試科目 (代碼)：工程數學 (3101)

共 2 頁，第 2 頁 *請在【答案卷】作答

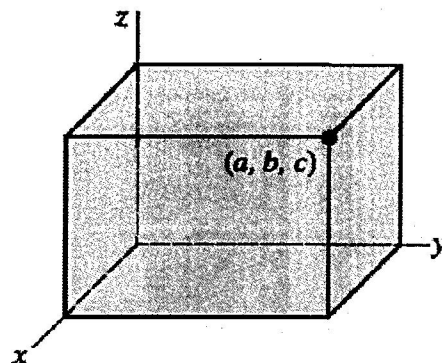
(c) Represent $f(x) = e^{-\alpha x}$, $\alpha > 0, x > 0$ by a cosine integral and by a sine integral. (5%)

(d) Determine the Fourier transform of $f(x) = \cos^2(2x) + \text{rect}(x/4)$.

Definition of Fourier transform: $\hat{f}(k) = \int_{-\infty}^{\infty} f(x)e^{ikx} dx$

Definition of rectangular function: $\text{rect}(x) = \begin{cases} 1 & |x| < 1/2 \\ 0 & \text{otherwise} \end{cases}$ (5%)

6. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in a rectangular parallelepiped shown in the figure with the right side ($y = b$) kept at temperature $g(x, z)$ and the left side ($y = 0$) kept at $f(x, z)$. The remaining sides are kept at temperature zero. (10%)



7. Apply Laplace transform to solve the displacement $u(x, t)$ of a string which is driven by an external force: (10%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \sin(\pi x) \sin(\omega t), \quad 0 < x < 1, t > 0$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0$$

$$u(x, 0) = 0, \quad \frac{\partial u}{\partial t} \Big|_{t=0} = 0, \quad 0 < x < 1$$

8. (a) Find the Laurent series of the complex function $f(z) = \frac{1}{z(z+4i)}$, $|z+2| > 3$

(5%)

(b) Determine the integral $\oint_C \left(z^2 \exp\left(\frac{1}{z}\right) + \frac{\sin z}{z^5 + 3z^3} \right) dz$ where C is the circle of

$\left| z - \frac{3i}{2} \right| = 2$, oriented counterclockwise. (5%)