注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：**聯合招生**

考試科目(代碼)：**電磁學**(9803)

一作答注意事項一

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。

2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。

3. 考生限在答案卷上標記「▟由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。

4. 答案卷用盡不得要求加頁。

5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。

6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：聯合招生 (0598)

考試科目（代碼）：電磁學 (9803)

電磁常數：　permittivity　　$\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m

　　　　　　permeability　　$\mu_0 = 4\pi \times 10^{-7}$ H/m

　　　　　　light speed　　　$c = 3 \times 10^8$ m/s

注意事項: 請以 SI 制單位回答下面所有問題

1. (10%) A capacitor consists of two concentric cylindrical surface, one radius a and another of radius b, as shown in the figure. The insulating layer is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2.
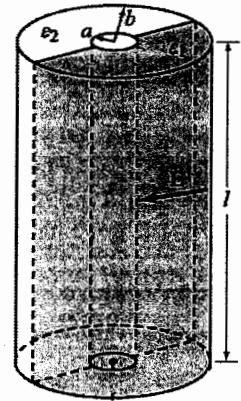
   (a) If the conducting surfaces are the two concentric cylindrical surface, calculate the capacitance C for $a = 1$mm, $b = 10$mm, $\varepsilon_1 = 10\varepsilon_0$, $\varepsilon_2 = 5\varepsilon_0$, and $\ell = 3$ cm.

   (b) Same as (a) but the conducting plates are on the top and bottom faces of the cylinder. Find C.

2. (15%) Charge Q is located at a distance h above a grounded half-plane in the x-y plane and at a distance d from another grounded half-plane in the x-z plane. Use the image method to
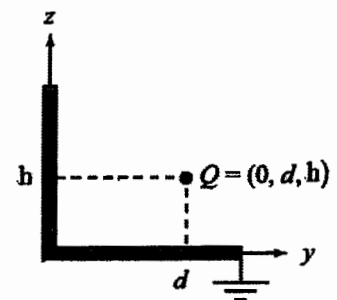
   (a) Establish the magnitudes, polarities, and locations of the images of charge Q with respect to each of the two ground planes (as if each is infinite in extent).

   (b) Find the electric potential V(x, y, z) and electric field E(x, y, z) at an arbitrary point P=(x, y, z)

3. (10%) A thin current element extending between $z = -L/2$ and $z = L/2$ carries a current I along +z direction through a circular cross-section of radius a.

   (a) Find the vector magnetic potential **A** at a point P=(x, y, z) located very far from the origin O. (We assume that $r = \sqrt{x^2 + y^2 + z^2} \gg L \gg a$)

   (b) Determine the corresponding magnetic field **H**.

4. (15%) Derive an expression for the magnetic field **H** at an arbitrary point P due to the current shown in the figure. If $z_1 = 2m$, $z_2 = 12m$, $z = 0$ and the current $I = 10A$, find **H** at $r= 10m$.
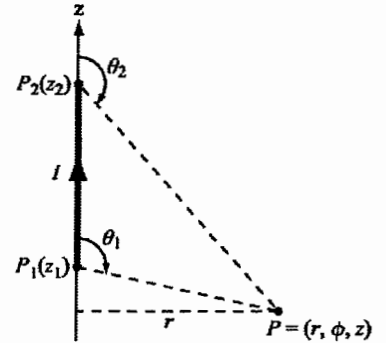
5. (10%) A loop (see figure) moves away from a wire carrying a current $I_1 = 10A$ at a constant velocity $\mathbf{u} = 8\hat{y}$ (m/s). If $R = 5\Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop.
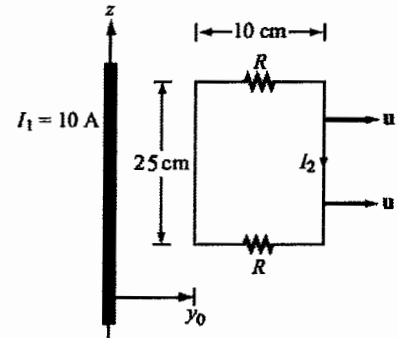
6. (10%) The electric field radiated by a short dipole antenna is given in spherical coordinates by $\mathbf{E}(r, \theta, \phi; t) = \hat{\boldsymbol{\theta}} \; \dfrac{5}{r} \sin\theta \cos(3\pi \times 10^9 t - 2\pi r)$ (V/m). Find $\mathbf{H}(r, \theta, \phi; t)$.

Formula: Curl **v** in spherical coordinates:

$$\nabla \times \mathbf{v} = \frac{1}{r \sin\theta}\left[\frac{\partial(\sin\theta\, v_\phi)}{\partial\theta} - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial(r v_\phi)}{\partial r}\right]\hat{\theta}$$
$$+ \frac{1}{r}\left[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

7. (10%) Dry soil is characterized by $\varepsilon_r = 2.5$, $\mu_r = 1$ and $\sigma = 10^{-4}$ S/m. At each of the following frequencies, determine if the dry soil may be considered a good conductor, a quasi-conductor or a low-loss dielectric; (a) 60Hz, (b) 1kHz, (c) 1MHz, (d) 1GHz.

8. (10%) A TE wave propagating in a dielectric filled waveguide of unknown permittivity has dimension $a = 10\text{cm}$ and $b = 6\text{cm}$. If the x-component of its electric field is given by $E_x = 10\cos(20\pi x)\sin(50\pi y)\sin(3.5\pi \times 10^9 t - 70\pi z)$ V/m, determine (a) the mode number, (b) ε_r of the material in the guide, (c) the cutoff frequency.

9. (10%) For some type of glass, the index of refraction varies with wavelength as $n = 1.75 - 0.1\lambda$ (λ in μm) where λ is the wavelength of light. Given that the wavelength of red light is 0.7μm and that of violet light is 0.4μm, determine the angular dispersion in degrees (as shown in the figure) if the incident white light is at an angle of 50° on the surface of a prism made of the glass.