1. (5%) Find the general solution for $\dfrac{d^2y}{dx^2} + y = 2cosx$

2. (5%) Find the Laplace transform for $f(t) = e^{2t}cost$

3. (5%) Find the Laplace transform for $f(t) = t - [t]$, where $[t]$ is the largest integer that is not larger than t.

4. (5%) Find the inverse Laplace transform for $\dfrac{1}{s^3+1}$.

5. (5%) $A = \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix}$, find e^{At} by using Cayley-Hamilton theorem.

6. (6%) Evaluate the following integrals.

    (a) (2%) $\int_0^i z^2\, dz$

    (b) (2%) $\oint_C e^z\, dz$, where C: $|Z| = 1$, clockwise.

    (c) (2%) $\oint_C \dfrac{1}{z-1} dz$, where C: $|Z| = 5$, clockwise.

7. (6%)

$$f(z) = \frac{1}{(z-1)(z-2)}$$

Integrate $f(z)$ counter clockwise, $\oint_C f(z)\, dz$, around C_1, C_2, and C_3.

    (a) (2%) C_1: $|z| = \dfrac{1}{2}$

    (b) (2%) C_2: $|z| = \dfrac{3}{2}$

    (c) (2%) C_3: $|z| = \dfrac{5}{2}$

參考用

注意:背面有試題

8. (6%)

$z = x + jy$

$f(z) = z^2$ 　　　, Calculate $f'(z)$ and $g'(z)$.

$g(z) = |z|^2$

9. (7%)

$f(z) = z^2 e^{\frac{1}{z}}$

find the Laurent Series of $f(z)$.

10. (10%)

$A = \begin{pmatrix} 8 & -6 \\ 4 & -2 \end{pmatrix}$

(a) (5%)$Ax = \lambda x$,λ is the eigenvalue of A, and x is the eigenvalue of A, Find λ and x.

(b) (5%) $A = SDS^{-1}$, S is the matrix with eigenvectors of A, and D is a diagonal matrix which is composed by the eigenvalues of A. Find S, D, and S^{-1}.

11. (15%) Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

(a) (3%)Find $rank(A)$.

(b) (4%)Find $N(A)$.

($N(A)$ is the nullity of A.)

(c) (4%)Find $dim(N(A)) + rank(A)$

($dim(N(A))$ is the dimension of $N(A)$)

(d) (4%)$Ax = b$, find x.

12. (12 %) Let V be a finite-dimensional vector space and $T: V \rightarrow V$ be linear. Suppose $rank(T) = rank(T^2)$. Prove that $R(T) \cap N(T) = \{0\}$.

注意:背面有試題

13. (13%) Given a vector space V over F. Define the dual space of $V^* \times V^*$ as the set of all function (also known as linear functional) from V to F, i.e., $V^* \stackrel{\text{def}}{=} \{f | f : V \to F\}$. It is obvious that V^* is itself also a vector space with the addition $+ : V^* \times V^* \to V^*$ and scalar multiplication $* : F \times V^* \to V^*$ define as pointwise addition as well as pointwise scalar multiplication. Given any linear transformation $T : V \to W$. The transpose T^t is a linear transformation from W^* to V^* defined by $T^t(f) = fT$ for any $f \in W^*$. For every subset S of V, we define the annihilator S^0 as $S^0 \stackrel{\text{def}}{=} \{f \in V^* | f(x) = 0 \ \forall x \in S\}$. Suppose V, W are both finite-dimensional vector spaces and $T : V \to W$ is linear. Prove that $N(T^t) = (R(T))^0$.