系所班組別：聯合招生 (0598)

考試科目（代碼）：電磁學 (9803)

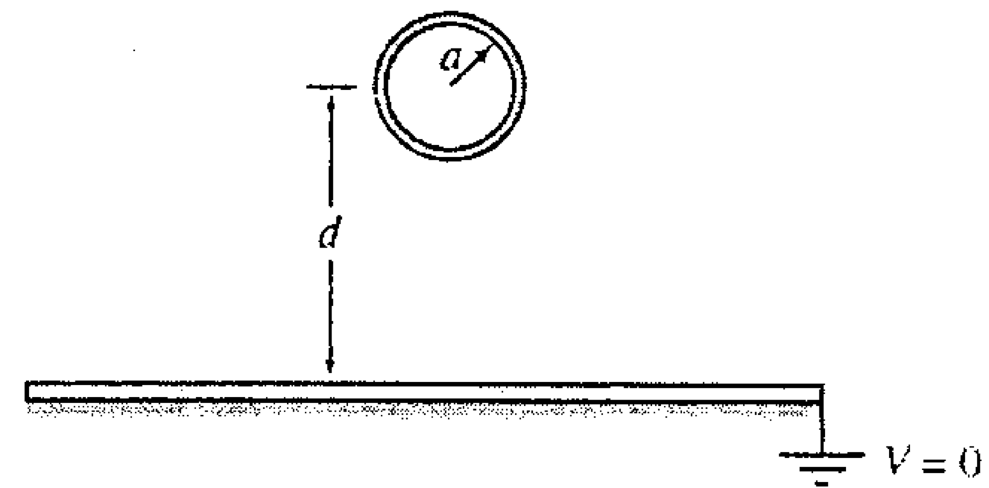
 *請在【答案卷】作答

請注意：1. 請在 MKS 制單位下回答所有問題

2. 電磁常數給於題目卷最後一頁

1. Write down the Maxwell's equation in matter for the electric field intensity vector $\vec{E}$, the electric flux density vector $\vec{D}$, the magnetic field intensity vector $\vec{H}$, and the magnetic flux density vector $\vec{B}$ in differential form and in integral form. (10%)

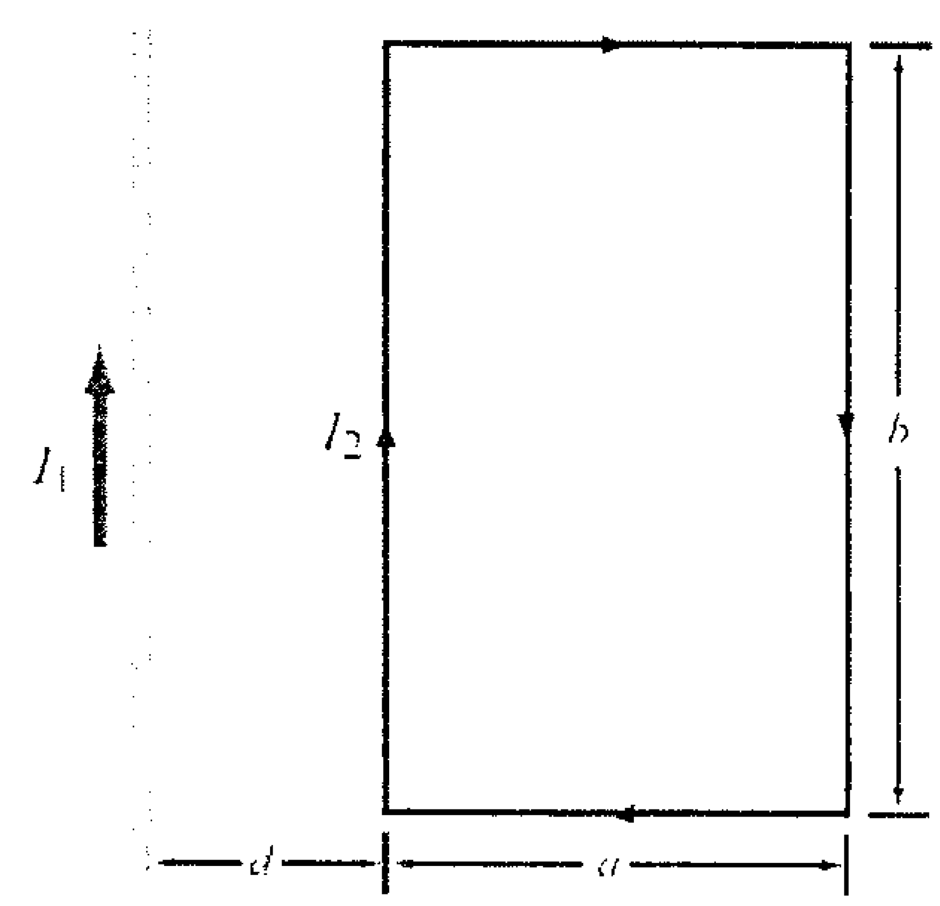
2. Find the capacitance per unit length C' of an infinitely long conducting cylinder of radius a situated at a distance d from a parallel conduction plane. (10%)



3. A coaxial capacitor consists of two concentric, conducting cylindrical surfaces, one of radius a and another of radius b, $(a < b)$.The insulting layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2. The length of the capacitor is ℓ. Find the capacitance C of the capacitor. (10%)

4. A long cylindrical conductor whose axis is coincident with the z-axis has a radius a and carries a current characterized by a current density $\vec{J} = J_0 e^{-2r}\hat{z}$, where J_0 is a constant and r is the radial distance from the cylinder's axis. Obtain an expression for the magnetic field $\vec{H}(r)$ for $0 \le r \le a$ and $r > a$. (10%)

5. A rectangular loop is coplanar with a long, straight wire in the air (see Figure). Determine the mutual inductance. (10%)



6. A parallel-plate capacitor is filled with a lossy dielectric material of relative permittivity ε_r and conductivity σ. The separation between the plates is d and each plate is of area A. The capacitor is connected to a time-varying voltage source $V(t) = V_0 \sin \omega t$.

(a) Obtain an expression for I_c, the conducting current flowing between the plates inside the capacitor. (3%)

(b) Obtained an expression for I_d, the displacement current flowing inside the capacitor. (3%)

(c) Give an equivalent-circuit representation for the capacitor. (4%)

7. (a) A circular loop of radius a carries a steady current I. Determine the magnetic field $\vec{H}$ at a point on the axis of the loop. (5%)

(b) Determine the magnetic field $\vec{H}$ at the middle point O of a finite solenoid, given that the solenoid is of length ℓ and radius a, and comprises N turns carrying current I. (5%)

8. In a nonmagnetic, lossy dielectric medium, a 250 MHz electromagnetic plane wave is characterized by the magnetic field phasor $\tilde{H} = (\hat{x} - j5\hat{z})e^{-4y}e^{-j9y}$ A/m. The electric field phasor can be expressed by $\tilde{E} = \tilde{\eta}(\hat{z} + j5\hat{x})e^{-4y}e^{-j9y}$ V/m. Determine $\tilde{\eta}$. Here $j = \sqrt{-1}$. (10%)

Note: The time domain fields are related to the phasor vectors by $\vec{E}(x,y,z,t) = Re\{\tilde{E}(x,y,z)e^{j\omega t}\}$, $\vec{H}(x,y,z,t) = Re\{\tilde{H}(x,y,z)e^{j\omega t}\}$.

9. A plane wave in the air (medium 1) with the electric field phasor $\tilde{E}_i = \hat{z}10e^{-j(3x+4y)}$ V/m is incident upon the planar surface of a dielectric material with $\varepsilon_r = 4$ (medium 2, nonmagnetic), occupying the half-space $x \geq 0$.

Determine (a) the angle of incidence θ_i, (b) the angular frequency ω, (c) the angle of transmission θ_t, (d) the time-domain expression for the transmitted electric field $\vec{E}_t$. (10%)

Hint: The Fresnel's transmission coefficient is $\tau_n = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$ for normal polarization and $\tau_p = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$ for parallel polarization where η_1 and η_2 are the intrinsic impedance of the medium 1 and 2, respectively.

10. The retarded time t_r is the time at which a field actually begins to propagate from a source point to an observer.

Consider a particle of charge q moving in a circle with the position given by $\vec{r}_{\mathrm{p}}(t) = \hat{x}R\cos\omega t + \hat{y}R\sin\omega t$.

(a) Calculate the retarded time t_r (from the charge particle) to an observer situated at a position $\vec{r} = \hat{z}z$ on the z-axis at time t. (4%)

(b) Calcuate the Lienard-Wiechert potentials on the z-axis, from the moving particle. (6%)

Note: Lienard-Wiechert potentials are the electric and magnetic potentials at the coordinate $(\vec{r}, t)$ generated by a moving point charge:

$V(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \frac{qc}{(\xi c - \vec{\xi} \cdot \vec{v}_p(t_r))}$, $\vec{A}(\vec{r}, t) = \frac{\vec{v}_p(t_r)}{c^2} V(\vec{r}, t)$ where $\vec{\xi} = \vec{r}(t) - \vec{r}_p(t_r)$ and $\xi = |\vec{\xi}|$. $\vec{v}_p(\dot{t_r})$ is the particle velocity at the retarded time t_r.

Constants (電磁常數)

Vacuum permittivity $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m

Vacuum permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

Light speed in vacuum $c_0 = 3 \times 10^8$ m/s