

國立清華大學 105 學年度碩士班考試入學試題

系所班組別：工程與系統科學系(0525) 乙組

考試科目（代碼）：工程數學 (2501)

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1. Find the interval of  $x$  for which the given ODE or IVP has an unique solution.

(a)  $\frac{1}{\ln x} \frac{dy}{dx} - y = 0$  (2%)

(b)  $\frac{d^2y}{dx^2} + \tan(x)y = e^x, y(0) = 1, y'(0) = 0$  (2%)

2. Obtain general solution for following ODE. (6%)

$$\frac{dy}{dx} = \frac{4x^2 + y^2}{xy}$$

3. Solve  $x \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} - y = 0$  about  $x = 0$ . (11%)

4. Solve following initial value problem: (8%)

$$\frac{d^2y}{dt^2} + y = g(t), \quad y(0) = 0, \quad y'(0) = 1$$

$$\text{where } g(t) = \begin{cases} 0, & 0 \leq t < \pi \\ 1, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$$

5. Solve following ODE for general solution: (6%)

$$\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} - 2x = 0$$

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6. (a) Find the the eigenvalues ( $\lambda_1 \geq \lambda_2 \geq \lambda_3$ ) of the matrix  $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}$  (5%)
- (b) Obtain an orthogonal matrix  $P$  which gives  $P^T A P = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ . (5%)

7. The position of a moving particle is given by  $\vec{r}(t) = (3 \cos t, 3 \sin t, 4t)$ . Find the unit tangent vector  $\vec{T}(t)$ , the unit normal vector  $\vec{N}(t)$ , and the binormal vector  $\vec{B}(t)$ . Find the curvature  $\kappa(t)$ . (5%)

8. (a) Evaluate  $\oint_C (x^7 + 8y)dx + (6x - e^y)dy$  on the curve  $C: (x - 5)^2 + (y + 3)^2 = 5$ , integrating along the positive direction. (5%)

- (b) Evaluate  $\iint_S (\vec{F} \cdot \vec{n}) dS$  where  $\vec{F} = (2x, 3y^2, 1 - 4z)$  and  $\vec{n}$  is the outward normal of the surface  $S$  of a cubic bounded by  $-1 \leq x \leq 3, 0 \leq y \leq 4, 5 \leq z \leq 9$ . (5%)

9. (a) Assume that  $f(x)$  is a  $2\pi$ -periodic function, defined in  $(-\infty, \infty)$ . Given

$$f(x) = \begin{cases} 1, & -\frac{\pi}{2} \leq x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \leq x < \frac{3\pi}{2} \end{cases} \quad \text{we expand } f(x) \text{ in Fourier series :}$$

$$f(x) = \sum_{n=0}^{\infty} a_n \cos(nx). \quad \text{Find } a_n \quad (n = 0, 1, 2, 3, \dots) \quad (5\%)$$

- (b) The differential equation  $5y'''' + 4y = f(x)$  can be solved by seeking the solution in the form  $y(x) = \sum_{n=0}^{\infty} d_n \cos(nx)$ , where  $f(x)$  is given in (a). Find  $d_n$  ( $n = 0, 1, 2, 3, \dots$ ) (5%)

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10. Let  $u(x, t)$  satisfy the equation (15%)

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < 1 \text{ and for } t > 0,$$

subject to the initial condition

$$u(x, 0) = f(x), \quad \text{for } 0 < x < 1,$$

and the boundary conditions

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=1} = -h u(1, t), \quad \text{for } t > 0,$$

where  $h$  is a positive constant. Solve for  $u(x, t)$ . It is required to show details of your work, including the *derivation* of the relevant orthogonal set of eigenfunctions to this problem.

11. Use Cauchy's residue theorem to evaluate the integral (15%)

$$\oint_C \frac{e^z}{1 - \cos z} dz,$$

where  $C$  is the *positively oriented* circle  $|z|=1$ . It is required to discuss (a) how

to locate and *classify* the singularity of the function  $f(z) = \frac{e^z}{1 - \cos z}$  within the

indicated contour  $C$ , and (b) how to find the corresponding residue.