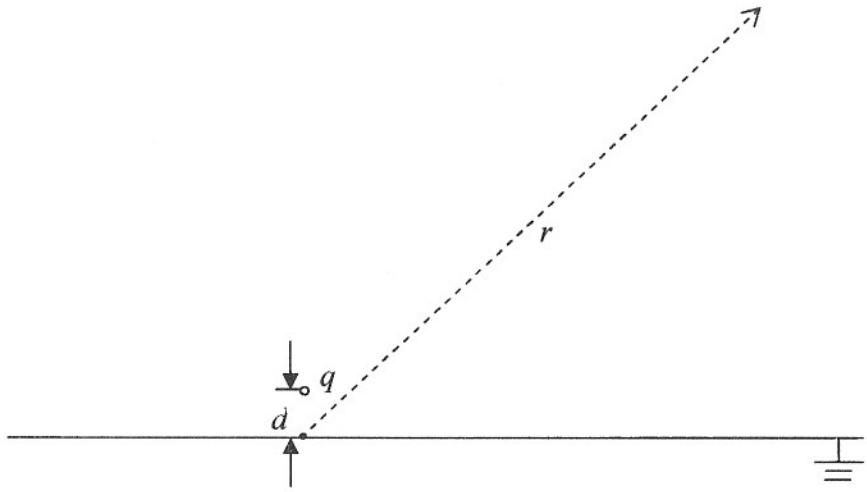
國　立　清　華　大　學　命　題　紙

96 學年度 電機領域聯合招生 系（所）＿＿＿＿＿＿ 組碩士班入學考試

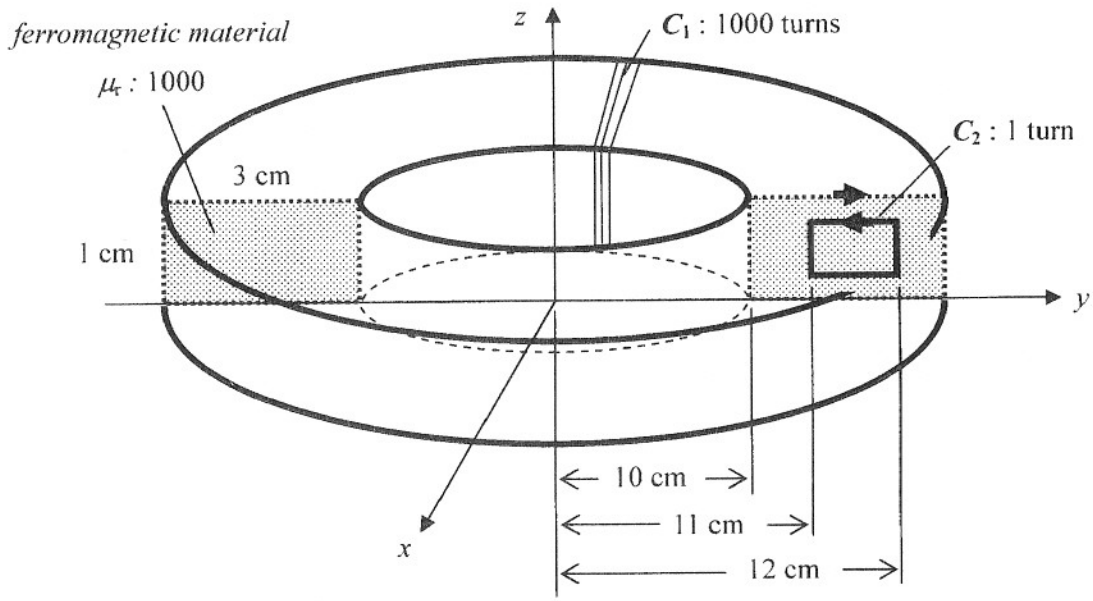
科目 電磁學 A 科目代碼 9906 共 4 頁第 1 頁 *請在【答案卷卡】內作答

1. (15%) (a) Determine the electric field everywhere induced by a charged dielectric sphere of radius R_0 and permittivity ε. The charge density $\rho(r)$ is distributed as a constant ρ_0 from origin to $R_0/2$ and $(\rho_0 R_0)/2r$ from $R_0/2$ to R_0. (b) If this sphere is concentrically enclosed in a conducting sphere of radius $2R_0$, what will be the electric field outside?

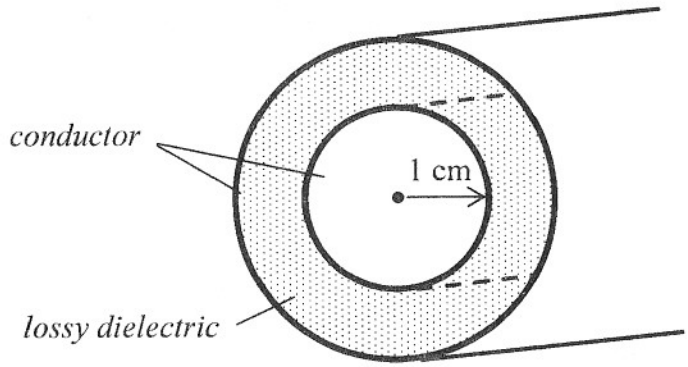
2. (10%) As shown in the following figure, a positive charge q is located d above a large conducting plate that is at potential zero. Show that the electric potential V in the far field ($r >> d$) is proportional to r^{-2}.



3. (15%) A tightly wound toroidal coil of 1000 turns, C_1, has an inner radius of 10 cm and a rectangular cross section with a width and a height of 3 cm and 1 cm, respectively, and the inside of the coil is filled with a ferromagnetic material of relative permeability $\mu_r = 1000$, as show in the figure below. A rectangular shaped single turn coil, C_2, with a width and a height of 1 cm and 0.5 cm, respectively, is placed inside the toroidal coil. Find **(a)** the mutual inductance between these two coils and **(b)** the net force on the single turn coil if the current on both coils is 1 A, flowing in opposite directions (as show by the arrows), and the ferromagnetic material is replaced by **air** ($\mu_r = 1$).



4. (10 %) A cylindrical coaxial cable consists of an outer and an inner conductor of radii 2 cm and 1 cm, respectively. A lossy dielectric material of dielectric constant 2.0 and conductivity, σ (S/m) = σ_0 / r, fills the space between the two conductors, where $\sigma_0 = 10^{-4}$ and r is the radial position (in unit of cm) from the center of the cable in cylindrical coordinates, as shown in the figure below.
   (a) Find the distribution of electrostatic potential, $V(r)$, between the conductors if a voltage of 10 V is applied on the inner conductor while the outer one being grounded ($V = 0$ V).
   (b) Find the resistance (between the conductors) per cm of this coaxial cable.



5. (15%) Electromagnetic waves in a linear, homogeneous and isotropic medium have the following scalar and vector potentials:

$$V = 0 \quad \text{and} \quad A = \frac{\mu_0 k}{8c}\exp(ct - |x|)\hat{z}$$

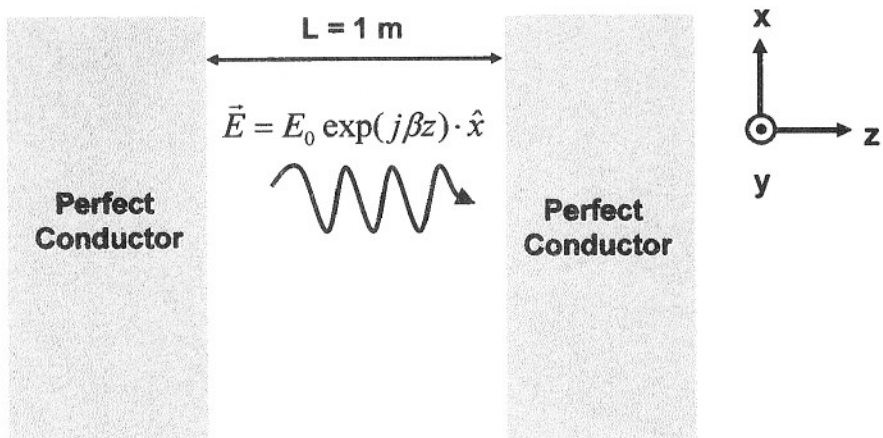
where k is a constant, $c = 1/\sqrt{\varepsilon_0 \mu_0}$, t is time, and $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors in the Cartesian coordinate system. Please answer the questions

(a) (5%) What are the electric and magnetic fields (including the direction of polarization)? Please draw figures to show the fields as a function of x.

(b) (5%) What are the source terms of carrier density (ρ) and current density (J)?

(c) (5%) Is there any surface charge or surface current at the position of $x = 0$?

6. (10%) A x-polarized uniform plane wave propagates in air in the $+z$ direction and impinges normally between two perfectly conducting mirrors which extend infinitely along x-y plane. Please answer the questions.



(a) (2%) If the distance between these two mirrors is 1 m, what are the frequencies such that the plane wave can resonate between these two mirrors?

(b) (4%) Please prove the time-averaged Povnting vector is equal to zero everywhere inside the mirror cavity at resonant condition. What is the stored energy density distribution?

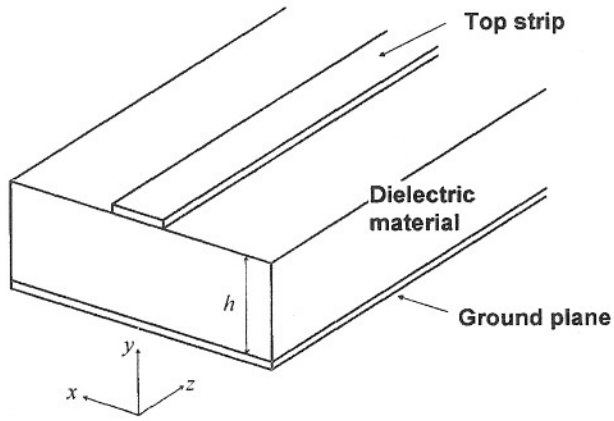
(c) (4%) What happens if this cavity is filled with ionized gases and the plane wave frequency is exactly the same as the cutoff frequency?

7. (15%) The figure below shows a microstrip line in a PCB board.

(a) (5%) Draw the electric field lines and magnetic field lines when the signal propagates along the z-direction.

(b) (5%) Draw the equivalent circuit model of the microstrip line.

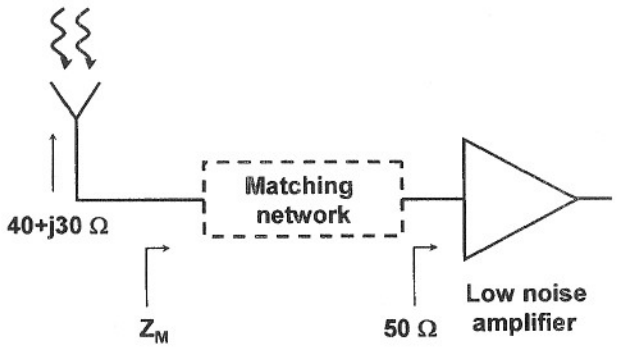
(c) (5%) When the substrate thickness h increases, how does the characteristic impedance Z_0 of the line change? Give a brief explanation based on the equivalent circuit model of the line.

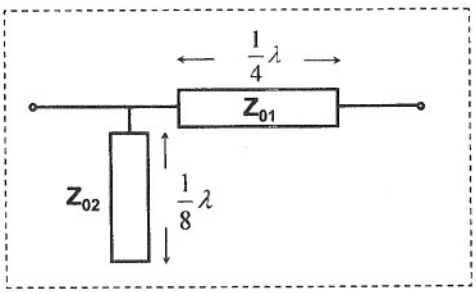


8. (10%) The figure below shows a typical RF front-end circuit composed of an antenna and a low noise amplifier:

(a) (5%) Explain why a matching network is essential between the antenna and the low noise amplifier. What is Z_M under a matched condition?

(b) (5%) Design a matching network based on the topology shown below. Determine Z_{01} and Z_{02}.





Matching network