九十一學年度 電子工程研究所 系（所） 組碩士班研究生招生考試

科目 近代物理 科號 2604 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Based on the uncertainty principle $\Delta x \Delta p \geq \frac{\hbar}{2}$

   (a) Calculate the minimum average kinetic energy of a particle with mass m confined in a limited region of space Δx.

   (b) Estimate the minimum kinetic energy of a particle with mass 0.2 Kg confined in a box of size 0.3 meter.

   The value of Planck constant is $6.626 \times 10^{-34} J \bullet s$. (15%)

2. (a) Find the wave function and the energy levels of a particle with mass m confined in a one-dimensional box of length d.

   (b) Find the probability that this particle can be found between 0.45d and 0.55d for the ground state.

   The following integral is given: $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4}$. (15%)

3. Consider a particle of mass M moving in a potential well of width L and depth V_0, with $V_0 >> \hbar^2 / (mL^2)$.

   (a) Estimate the ground state energy. (8%)

   (b) Estimate the tunneling depth of the ground state particle into the barrier. (7%)

4. In a 3-dimensional spherically symmetric system, such as the case of a Hydrogen atom, the orbitals have angular momentum quantum number l, and possible electron wave functions are S for $l = 0$ and P_x, P_y and P_z for $l = 1$, respectively.

   (a) Draw the probability distributions of S and P_x qualitatively. (7%)

   (b) Now, if a weak external electric field +1(in certain appropriate units) is applied along the x-axis, the above orbitals will be slightly distorted. Draw the distorted probability distributions of S and P_x, assuming the particle is an electron. (8%)

   (c) The orbital energies will also be shifted by the above electric field. Compare the energy shifts of P_x and P_y due to the field. Which one is larger? And why? (5%)

5. An electron in semiconductor sees a complicated crystal potential; however, to first order approximation, the effect of this complicated crystal potential can be taken into account by modifying the mass of the electron; that is, an electron in the crystal potential can be regarded as a *free* electron with an effective mass m^*. Suppose you are now given a certain kind of semiconductor, in which the effective mass of electron, m^*, is $0.1m_0$, where m_0 is the mass of a free electron in vacuum, and the dielectric constant $\varepsilon_r=10$. We also know that the electron in the semiconductor around a positive donor ion experiences a Coulomb potential $V(x) = -\frac{q^2}{4\pi\varepsilon_r\varepsilon_0 x}$.

   (a) Write down the Schrodinger equation for the electron in the semiconductor around the positive donor ion. (4%)

   (b) Write down the dimensions (units) of the coefficients of each terms in the Schrodinger equation, and discuss the dependence of energy on m^* and ε_r. (10%)

   (c) We know that the ground state energy of the hydrogen atom is -13.6 eV. Please estimate the binding energy of the electron around the donor ion in the semiconductor. (5%)

   (d) Similar to (b) and (c), discuss the dependence of *length* on m^* and ε_r, and estimate the Bohr radius of the electron in the ground state. You are given that the Bohr radius of the electron in the ground state energy of the hydrogen atom is $0.53\,\mathring{A}$. (6%)

   (e) Plot qualitatively the wave function of the 6^{th} states for this electron around the donor ion. Please pay attention to the number of nodes, the distance between the nodes, and the amplitude. (10%)