1. Consider two electric dipoles $\vec{p}_1$, $\vec{p}_2$, and a charge q, lying at positions $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_q$, respectively. Which of the following statements are true? (複選) (多勾者倒扣, 扣完爲止) (15%)

(a) The electric potential energy of the dipole $\vec{p}_1$ is $(\vec{p}_1 \cdot \vec{E}_1)$, where $\vec{E}_1$ is the electric field at $\vec{r}_1$.

(b) The electric field at $\vec{r}_1$ is $\vec{E}_1 = -\nabla_{\vec{r}_1}(V_1)$, where $V_1(\vec{r}_1) = q/|\vec{r} - \vec{r}_1| + V_{12}$, V_{12} being the electric potential at $\vec{r}_1$ due to the dipole $\vec{p}_2$.

(c) The potential energy between q and $\vec{p}_2$ is $V_{q2} = q[\vec{p}_2 \cdot (\vec{r}_2 - \vec{r}_q)]/|\vec{r}_2 - \vec{r}_q|^3$.

(d) The electric potential at q due to dipole $\vec{p}_2$ is $-[\vec{p}_2 \cdot (\vec{r}_2 - \vec{r}_q)]/|\vec{r}_2 - \vec{r}_q|^3$.

(e) The electric potential at $\vec{r}_1$ due to the dipole $\vec{p}_2$ is $V_{12} = -\vec{p}_2 \cdot (\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|^3$.
(f) None of the above is true.

2. Consider the following magnetic field

$$\vec{B}(x,y,z) = \begin{cases} (0, B_0, 0), & 0 \le z \le L \\ 0, & eslewhere \end{cases}$$

, where B_0 is a constant. Let $\vec{A}(x,y,z)$ be the corresponding vector field. Which of the following statements are true? (複選) (多勾者倒扣, 扣完爲止) (10%)

(a) There are many choices for $\vec{A}(x,y,z)$ as long as they satisfy $\vec{B}(x,y,z) = \nabla \times \vec{A}(x,y,z)$.

(b) We can choose $\vec{A}(x,y,z) = 0$ for $|z| > L$.

(c) A possible choice for $\vec{A}(x,y,z)$ is the following.

$$\vec{A}(x,y,z) = \begin{cases} 0, & z < 0 \\ (B_0 z, 0, 0), & 0 \le z \le L \\ 0, & z > L \end{cases}.$$

(d) A possible choice for $\vec{A}(x,y,z)$ is the following.

$$\vec{A}(x,y,z) = \begin{cases} 0, & z < 0 \\ (B_0 z, 0, 0), & 0 \le z \le L \\ (B_0 L, 0, 0), & z > L \end{cases}$$

(e) None of the above is true.

3.

(a) A very long and very thin straight wire located along the z-axis carries a current I in the z-direction. Find the magnetic field intensity at any point in free space using Ampere's law in integral form. (5%)

(b) Does the equation of continuity imply charge conservation? How? (5%)

(c) Find electric field intensity due to an isolated point charge q. Starting from the differential form of the Gauss's law. (5%)

(d) Explain the working principle of a lightning arrestor (避雷針). (5%)

(e) Find the inductance per unit length of a very long solenoid having n turns per unit length. The permeability of the core is μ. The magnetic flux density of the solenoid is μnI. (5%)

4. The electric field intensity of a time-harmonic plane electromagnetic wave is given by

$\bar{E}(x,y,z,t) = \hat{a}_x 126 \times \sin[10^9 t - 6y + 8z - 0.2]$ Volt/m, where all the variables are in MKSA units. This wave propagates in a simple, nonmagnetic medium. Find numerically, including units if applicable, for

(a) the unit vector pointing to the wavefront propagation direction, (Indicate the direction of the vector by plotting it in the y-z plane). (3%)
(b) the wavelength along the wavefront normal direction, (3%)
(c) the wavefront velocity or the phase velocity of the electromagnetic wave in the wavefront propagation direction, (3%)
(d) the phase velocity in the z direction, (3%)
(e) the wavelength in the y direction, (3%)
(f) the refractive index of the medium, (3%)
(g) the wave impedance, (3%)
(h) the time-average intensity (power per unit area) of the wave. (4%)

Hint: the vacuum permittivity is $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m.

and the vacuum permeability is $\mu_0 = 4\pi \times 10^{-7}$ H/m

5. Consider a short dipole antenna of length ℓ carrying a current $I_0 \cos(\omega t)$ located at origin and oriented along the z axis in free space. The electric field component of the wave at distance r very much greater than the wavelength is approximately given by

$$\bar{E}(R,\theta) = \hat{a}_\theta j \frac{30\beta I_0 \ell}{R} \sin\theta e^{-j\beta R} \qquad \text{where } \beta = \omega\sqrt{\mu_0 \varepsilon_0}$$

(a) Find the corresponding magnetic field $\bar{H}(R,\theta)$ (5%)

(b) Find the time averaged poynting vector $\vec{P}_{av}$ (5%)

(c) Find the total power radiated by the dipole source by integrating the poynting vector over a spherical surface centered at the dipole. (5%)

(d) Show that the radiation resistance of this short dipole can be expressed as

$$R_{rad} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2 \quad \text{where } \lambda \text{ is the wavelength.}$$

(5%)

(e) What is the total power radiate from a short antenna of $\ell = 0.01\lambda$ excited with 1 A current. (5%)

Note that the wave impedance is $120\pi\Omega$ here.

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix}\mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & A_z\end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial\phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix}\mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta\\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ A_R & RA_\theta & (R\sin\theta)A_\phi\end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$

$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$

$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$