國立清華大學命題紙
甲丝(藝)、紅色(製計、製造組)
99學年度 動力機械工程學系丙組(固體與奈微米力學組) 碩士班入學考試

科目 工程數學 科目代碼 1003 共 3 頁,第 1 頁 *請在【答案卷卡】作答

1. Solve the following ordinary differential equations

(1a)
$$y' = 2 + \sqrt{y - 2x + 3}$$
 (7%)

(1b)
$$4y'' - 4y' + y = e^{x/2}\sqrt{1-x^2}$$
 (8%)

2. Solve the following pair of simultaneous differential equations by Laplace transform

$$\frac{d^2x}{dt^2} - x = y$$
$$\frac{d^2y}{dt^2} + y = -x$$

given that at t = 0, x = 2, y = -1, $\frac{dx}{dt} = 0$, and $\frac{dy}{dt} = 0$ (15%)

3.

(3a) Consider a linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$

Find the QR decomposition of the matrix A and the least squares solution of the above system.

(3b) Let the eigenvectors be v1, v2, and v3, describing the general solution to the differential equation

$$\frac{du}{dt} = \mathbf{K}u, \text{ where } \mathbf{K} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}; \text{ Find the eigenvalues of the matrix } \mathbf{K}. (5\%)$$

(3c) At what time T is the solution u (T) guaranteed to equal its initial value u (0)? (5%)

國立清華大學命題紙 中組(藝滋組)、丁紹(設計、製造生組) 動力機械工程學系丙組(固體與奈微米力學組) 碩士班入學考試

科目<u>工程數學</u>科目代碼<u>1003 共 3</u> 頁 第 <u>2</u> 頁 *請在【答案卷卡】作答 0803、1103

- 4. Given a vector function $\vec{E} = \vec{i}xy + \vec{j}(3x y^2)$, and the coordinates of three points $P_1(5, 6)$, $P_2(5, 3)$ and $P_3(3, 3)$, where \vec{i} and \vec{j} are unit vectors along x- and y- axes respectively.
 - (4a) Evaluate the integral $\sqrt{\overline{E} \cdot d\overline{l}}$ from P₁ straight to P₃. (5%)
 - (4b) Evaluate the integral $\int \overline{E} \cdot d\overline{l}$ from P₁ to P₃ along the piecewise straight path P₁ P₂ P₃, i.e. integrate from P₁ along straight-line segment to P₂ and then along another straight-line from P₂ to P₃. (5%)
 - (4c) Is this \overline{E} a conservative field? And why? (5%)
- 5. Consider the following partial differential equation for the function $\varphi(\xi,t)$

$$\omega_0^2 \varphi_{\xi\xi} - \varphi_{tt} = 0$$

The boundary conditions are:

$$\omega_0^2 \varphi_{\varepsilon}(0,t) - \varepsilon \varphi_{tt}(0,t) = 0$$

$$\omega_0^2 \varphi_{\xi}(1,t) + \varepsilon \varphi_{tt}(1,t) = 0$$

In the above equations, ω_0 and ε are constants.

- (5a) Assume that $\varphi(\xi,t) = \Phi(\xi)T(t)$, find the time free behavior equation $\Phi(\xi)$ and the boundary conditions by the method of separating variables or the product method. (3%)
- (5b) Write the eigenvalue equation of the system from the time free behavior equation and boundary conditions. (3%)
- (5c) Find the complete eigenvalues, when $\varepsilon = 0$. (3%)
- (5d) From the first nonzero eigenfunction $\Phi_1(\xi)$, find the behavior equation with time $T_1(t)$ and the corresponding solution $\varphi_1(\xi,t)$ (3%)

國立清華大學命題紙
甲組(藝內流組)、丁組(設計、製造組)
動力機械工程學系丙組(固體與奈微米力學組) 碩士班入學考試

6. Consider the following ordinary differential equation

$$\ddot{y} + 2D\dot{y} = x(t)$$

where D is a constant. The right hand side is given by

$$\mathbf{x}(t) = \begin{cases} 0 & t < 0 \\ \exp(-2Dt) & t \ge 0 \end{cases}$$

Solve the equation by the method of Fourier transformation:

$$Y(i\omega) = F(i\omega) \cdot X(i\omega)$$

- (6a) Determine the complex input spectrum X(iω). (2 %)
- (6b) Find the relationship in the complex form of the system F(iω) and X(iω) (3 %)
- (6c) Find the complex output spectrum Y(iω) (3 %)
- (6d) Calculate in the spectrum density $S(\omega)=|Y(i\omega)|$ and the value S(0) and $S(\infty)$. (2%)
- (6e) Calculate the solution in the time domain y(t) for t>0 through the inverse Fourier transform. (3%)

Remark:

$$\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{i\omega(a+i\omega)^2} d\omega = \begin{pmatrix} 0 & , t < 0 \\ \frac{2\pi \cdot \gamma(2,at)}{a^2 \cdot \Gamma(2)} & , t > 0 \end{pmatrix}$$

with
$$\Gamma(2) = \int_0^\infty te^{-t} dt$$
, $\gamma(2, at) = \int_0^{at} te^{-t} d\bar{t}$,
$$\int xe^{-x} dx = \frac{-(x+1)}{e^x}$$

7 Find all Laurent series of

$$\frac{1}{z^2 + 2z}$$

with center z = 0, and indicate their respective regions of convergence. (9%)

8 Evaluate the following integral

$$\int_{0}^{\infty} Im(z)dz$$

where C is the unit circle around z = 0 in the complex plane. (6%)