

國立清華大學命題紙

97 學年度 動力機械工程 系(所) 甲、丙、丁 組碩士班入學考試

1103, 1203

科目 工程數學 科目代碼 1003 共 2 頁第 1 頁 \*請在【答案卷卡】內作答

1. Find a general solution of

(i)  $\frac{dy}{dx} - xy = \frac{x}{y}$  (10%)

(ii)  $\frac{d^2y}{dx^2} + 4y = \sec 2x$  (10%)

2. Solve the following initial value problem

$$\frac{d^2y}{dt^2} + 4y = \begin{cases} 0 & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ 0 & t > 2\pi \end{cases}$$

with  $y = 0$  and  $\frac{dy}{dt} = 2$  at  $t = 0$ . (10%)

3. Determine clearly all the nature (real symmetric, anti-symmetric, Hermitian, orthogonal or unitary) of the following matrices. (Each matrix may contain more than one nature. No proof is needed.) (10%)

(A)  $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ ; (B)  $\begin{bmatrix} 2 & 1-i \\ 1+i & 5 \end{bmatrix}$ ; (C)  $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ ; (D)  $\begin{bmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ ;

(E) The matrix =  $\begin{bmatrix} 1-\cos \theta & -\sin \theta \\ \sin \theta & 1-\cos \theta \end{bmatrix} \begin{bmatrix} 1+\cos \theta & \sin \theta \\ -\sin \theta & 1+\cos \theta \end{bmatrix}^{-1}$

4. Let  $\phi(x, y, z)$  and  $\varphi(x, y, z)$  be continuous with continuous first and second partial derivatives on a smooth closed surface  $\Sigma$  and its interior  $M$ . Suppose both  $\nabla\phi = \vec{0}$  and  $\nabla\varphi = \vec{0}$  in  $M$ . Prove that  $\iiint_M (\phi\nabla^2\varphi - \varphi\nabla^2\phi) dV = 0$ . (hint: Gauss's divergence theorem) (10%)

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1103、1203

5. Find the complex Fourier integral of  $f(x) = x \exp(-|x|)$ . (10 %)

6. (i) Show that the following partial differential equation

$$\frac{\partial u}{\partial t} = k \left( \frac{\partial^2 u}{\partial x^2} + A \frac{\partial u}{\partial x} + Bu \right) \quad \text{where } k, A \text{ and } B \text{ are constants}$$

can be transformed into a simplified equation like  $\frac{\partial v}{\partial t} = k \frac{\partial^2 v}{\partial x^2}$  by choosing  $\alpha$  and  $\beta$

appropriately and letting  $u = v \exp(\alpha x + \beta t)$ . (10 %)

(ii) Use the previous idea to solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial u}{\partial x} + 2u$$

$$u(0, t) = u(\pi, t) = 0 \quad \text{for } t \geq 0$$

$$u(x, 0) = x(\pi - x) \quad \text{for } 0 \leq x \leq \pi \quad (10 \%)$$

7. (i) Please use contour integration to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^2(2+2x+x^2)} dx. \quad (10 \%)$$

(ii) Indicate true or false for each of the following statements about complex

variables. (No proof is needed. The wrong answer will be given no score but will be deducted 2 points. 每小題答錯倒扣 2 分) (10 %)

(A) The value of  $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$  does not exist;

(B) If  $f(z) = xy^2 + ix^2y$ , then  $\frac{df(z)}{dz}$  and  $f(z)$  are analytic at  $z = 0$ ;

(C)  $f(z) = (e^x \cos y) + i(e^x \sin y)$  is an analytic function;

(D) If  $f(z)$  is analytic, then  $\int_{z_1}^{z_2} f(z) dz = - \int_{z_2}^{z_1} f(z) dz$ ;

(E) A unit disk in  $z$  plane is mapping onto the upper half of  $w$  plane

via the transformation  $w = \frac{i-z}{i+z}$ .