QUESTION 1 (20%)

(a) Show that after making the dimensionless transformation

$$\theta = \frac{T - T_0}{T_w - T_0}, \quad \eta = \frac{x}{\delta(t)}$$

the one-dimensional transient heat conduction problem

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, \quad T(x,0) = T_0, \quad T(0,t) = T_w, \quad T(\infty,t) = T_0$$

becomes

$$\frac{d^2\theta}{d\eta^2} + \left(\frac{\delta}{\alpha}\frac{d\delta}{dt}\right)\eta\frac{d\theta}{d\eta} = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0$$

(b) The transformed equation in (a) would becomes an ordinary differential equation for the solution $\theta(\eta)$ if the function $\delta(t)$ is properly defined such that the coefficient $(\delta/\alpha)(d\delta/dt)$ is a constant, say $(\delta/\alpha)(d\delta/dt) = 1$. This particular transformation is known as "similarity transformation." Determine the similarity variable $\eta(x,t)$.

(c) Solve the problem

$$\frac{d^2\theta}{d\eta^2} + \eta\frac{d\theta}{d\eta} = 0, \quad \theta(0) = 1, \quad \theta(\infty) = 0$$

(d) Suppose $\theta(\eta_1) = 0.01$. Determine the penetration thickness, i.e. the thickness of the region where $0.01 \le \theta \le 1$.

QUESTION 2 (10%)

Forced convection on Blasius flow (boundary layer flow along a flat plate) with uniform wall temperature has the local Nusselt number

$$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

for Prandtl number of $0.6 \le \mathrm{Pr} \le 50$. The free stream velocity and temperature are U_∞ and T_∞, while $\mathrm{Re}_x = \rho U_\infty x/\mu$ is the local Reynolds number.

(a) Find the average Nusselt number $\overline{Nu}_L$ over the region $0 \le x \le L$.

(b) What is the overall heat transfer rate needed to maintain the wall temperature $T(x,0) = T_w$ in the region $0 \le x \le L$?

QUESTION 3 (20%)

Consider the heat transfer problem associated with a circular pipe of internal diameter D, heated under a uniform heat flux q''. The inlet flow is at a uniform temperature T_0 and a uniform velocity U_0. The temperatures at the internal wall surface and the pipe center are T_w and T_c, respectively. The mean fluid temperature at a local position x is $T_m(x)$.

(a) How do we define the local convection heat transfer coefficient $h(x)$ in terms of q'' and a suitable temperature difference? How do we define the local Nusselt number in terms of $h(x)$ and other suitable parameters? (8%)

(b) Schematically (示意) draw $h(x)$ versus x from the pipe inlet up to some position in the fully developed region. Also explain for the trend of its variation. (8%)

(c) Schematically draw $T_w(x)$ and $T_m(x)$, on a single diagram, versus x from the pipe inlet up to some position in the fully developed region. (4%)

QUESTION 4 (20%)

Solar energy resource becomes "hot" in recent years.

(a) Draw a schematic diagram for the spectral emissive power $E_\lambda(\lambda)$ versus the wavelength λ (μm) in log-log coordinates to show how the solar energy distributes with respect to λ. Consider the sun as a black emitter at a temperature of 5800 K. (8%)

(b) In what wavelength region is the spectral emissive power maximum? What do we name this range of wavelength? (6%)

(c) How do we define the total emissive power E in terms of the spectral emissive power $E_\lambda(\lambda)$? Write the expression for the total emissive power per unit area of the sun E_{sun}. (6%)

QUESTION 5 (14%)

Spin plays an important role in the flight trajectory of golf, ping-pong and tennis balls. Therefore, it is important to know the rate at which spin decreases for a ball in flight. The aerodynamic torque, T, acting on a ball in flight, is thought to depend on flight speed, V, air density, ρ, air viscosity, μ, ball diameter, D, spin rate (angular velocity), ω, and diameter of the dimples on the ball, d. determine the dimensionless parameters that result. Please also discuss briefly how the parameters affect the motion of the ball.

QUESTION 6 (8%)

How fast will a toy balloon filled with helium rise through still air? List all of your assumptions.

QUESTION 7 (8%)

Starting with the enthalpy form of the energy equation, show that for isentropic flows, the stagnation temperature remains constant.