# 國 立 清 華 大 學 命 題 紙

96 學年度動力機械工程學系（所）甲、乙、丙、丁 組碩士班入學考試

科目 工程數學 科目代碼 1003, 1103, 1203, 1303 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. Use the Laplace transform to solve the given initial-value problem

$$y''' - 3y'' + 3y' - y = t^2 e^t, \quad y(0) = 1, y'(0) = 0, y''(0) = -2 \qquad (20\%)$$

2 . Suppose A, B, and C are all matrices, indicate true or false for each of the following statements. (No proof is needed.)

(a) If AB=0 then A=0 or B=0;

(b) If $A^2$=0 then A=0;

(c) (A+B)(A – B)=$A^2 - B^2$;

(d) If AB=AC and A is non-invertible, then B=C; (6%)

(e) All the eigenvalues of a n × n Hermitian matrix are real;

(f) If A is a n × n unitary matrix and its eigenvalue is $\lambda$, then $|\det(A)|$=1 and $|\lambda|$=1.

3. A bar of length L is perfectly insulated both at the ends x=0 and x=L. The initial temperature distribution at the bar is u(x, 0) = f(x). Physical information: the flux of heat through the faces at the ends is proportional to the values of $\partial u / \partial x$ there. The governing equation is $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

(a) Show the temperature distribution $u(x,t)$ within the bar using separation of variables. (15%)

(b) If $f(x) = 1 + \cos(2\pi x / L)$, what are the final temperatures at x=0, L/2 and L as $t \to \infty$? (5%)

4. (a) Given a vector $\vec{A}=A_x\vec{a_x}+A_y\vec{a_y}+A_z\vec{a_z}$, where $\vec{a}_x$, $\vec{a}_y$, $\vec{a}_z$ are the unit vectors in $x$, $y$, $z$ directions, respectively. Express Curl $\vec{A}$ $(\vec{\nabla}\times\vec{A})$ and Div $\vec{A}$ $(\vec{\nabla}\bullet\vec{A})$ in terms of $A_x$, $A_y$ and $A_z$. (6%)

(b) Given a vector function $\vec{F}=y\vec{a_x}+x\vec{a_y}$, evaluate the scalar line integral $\int\vec{F}\bullet d\vec{l}$ from $P_1(2,1,-1)$ to $P_2(8,2,-1)$ along the straight line joining the two points. (14%)

5. (a) Find the inverse Laplace transform of the function

$$F(s)=\frac{s^2-5s+4}{s\left(s^2+1\right)}$$ (10%)

(b) Find the Laplace transform of the function

$$g(t)=\begin{cases} 2t; & t<3 \\ 1; & t\geq 3 \end{cases}$$ (10%)

6. Please use contour integration to evaluate the integral $\int_0^\infty \frac{x^2}{1+x^4}dx$. (14 %)