94 學年度 動力機械工程學 系（所） 乙 組碩士班入學考試

科目 控制系統 科目代碼 1601 共 8 頁第 1 頁 *請在試卷【答案卷】內作答

第一部份、電腦閱卷

● 以下十三題是單選題，請將答案填寫在電腦閱卷卡上

(1) Consider the discrete signal $x^*(kT)$ whose z-transform is

$$X(z) \triangleq \mathcal{Z}\left[x^*(kT)\right] = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-aT}z^{-1}}, \quad a>0$$

What is the final value of $x^*(kT)$ at $k \to \infty$? (2%)

(A) 1, (B)0, (C) -1, (D) $\frac{1}{1-e^{-aT}}$, (E) $\frac{-1}{1-e^{-aT}}$

(2) Consider the discrete signal $x^*(kT)$, where

$$x^*(kT) = \begin{cases} e^{-aT} & k \geq 0 \\ 0 & k < 0 \end{cases}$$

What is the z-transform of $x^*(kT)$? (2%)

(A) $\frac{z}{1-e^{-aT}}$, (B) $\frac{1}{z-e^{-aT}}$, (C) $\frac{z}{z-e^{-aT}}$, (D) $\frac{z}{z-a}$, (E) $\frac{1}{z-a}$

(3) Consider the discrete signal $x^*(kT)$ whose z-transform is

$$X(z) \triangleq \mathcal{Z}\left[x^*(kT)\right] = \frac{10z+5}{(z-1)(z-0.2)}$$

What is the value of $x^*(kT)$ at $k=3$? (2%)

(A)0, (B)10, (C)17, (D)18.4, (E)18.68

(4) Which of the following is a stable discrete system? (2%)

(A) $\frac{10(z-2)}{(z-1.1)(z+0.9)}$, (B) $\frac{10(z-2)}{(z-0.1)(z+1.9)}$, (C) $\frac{10(z-2)}{(z-0.1)(z+0.9)}$

(D) $\frac{10(z-0.2)}{(z-1.1)(z+0.9)}$, (E)None of them

(5) Consider the system defined by the following state-space description

$$\dot{x}(t)_{n\times1} = Fx(t) + Gu(t)_{1\times1}$$
$$y(t)_{1\times1} = Hx(t) + Ju(t)$$

When is this system completely controllable? (2%)

(A) $F=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix},\ G=\begin{bmatrix}1\\ 0\end{bmatrix},\ H=\begin{bmatrix}11 & -8\end{bmatrix},\ J=0$

(B) $F=\begin{bmatrix}1 & 1\\ 2 & -1\end{bmatrix},\ G=\begin{bmatrix}0\\ 1\end{bmatrix},\ H=\begin{bmatrix}1 & -18\end{bmatrix},\ J=0$

(C) $F=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix},\ G=\begin{bmatrix}1\\ 0\end{bmatrix},\ H=\begin{bmatrix}1 & -18\end{bmatrix},\ J=0$

(D) $F=\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix},\ G=\begin{bmatrix}0\\ 1\end{bmatrix},\ H=\begin{bmatrix}11 & -8\end{bmatrix},\ J=0$

(E) None of them

(6) Further to Problem (5), when is this system completely observable? (2%)

(A) $F=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix},\ G=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},\ H=\begin{bmatrix}4 & 5 & 1\end{bmatrix},\ J=0$

(B) $F=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix},\ G=\begin{bmatrix}12\\ -4\\ 1\end{bmatrix},\ H=\begin{bmatrix}4 & 5 & 1\end{bmatrix},\ J=0$

(C) $F=\begin{bmatrix}0 & 1 & 0\\ 0 & 8 & 1\\ -6 & -11 & -6\end{bmatrix},\ G=\begin{bmatrix}12\\ -4\\ 1\end{bmatrix},\ H=\begin{bmatrix}4 & 5 & 1\end{bmatrix},\ J=0$

(D) $F=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix},\ G=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},\ H=\begin{bmatrix}1 & 5 & 4\end{bmatrix},\ J=0$

(E) None of them

(7) Further to Problem (5), assume that the system is

$$F=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -2 & -3\end{bmatrix},\ G=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},\ H=\begin{bmatrix}1 & -5 & 4\end{bmatrix},\ J=0$$

which of the following state feedback gain will result in a close-loop system with three poles at $s=-10,\ -2\pm j3.464$? (2%)

(A) $K=[11\quad 54\quad 160]$, (B) $K=[160\quad 11\quad 54]$, (C) $K=[54\quad 160\quad 11]$,

(D) $K=[160\quad 54\quad 11]$, (E) None of them

(8) Further to Problem (5) and assume that

(i) Eigen-values of F are： $+5,\ +1,\ -2.4,\ -3\pm 6j,\ -43\pm 28j,\ -300\pm 528j$

(ii) $\det\begin{bmatrix} sI-F & -G \\ H & J \end{bmatrix}=0$ for $s=+1,\ -8,\ -300\pm 528j$

(iii) $\text{rank}(C_X)=9$, C_X is the controllability matrix,

(iv) $\text{rank}(Q_X)=6$, Q_X is the observability matrix,

How many system poles are there in this plant? (2%)

(A)3, (B)5, (C)6, (D)7, (E)9

(9) Further to Problem (8), how many system zeros are there in this plant? (2%)

(A)3, (B)5, (C)6, (D)7, (E)9

(10)Further to Problem (8), which of the following observer gain leads to an asymptotically stable observer? (2%)

(A) $L=[1\quad -3\quad 2\quad 14\quad -38\quad 6]$, (B) $L=[1\quad -13\quad 20\quad -41\quad 8\quad 0]$,

(C) $L=[0\quad -14\quad 22\quad -3\quad 8\quad -31]$, (D) $L=[51\quad 0\quad -32\quad -14\quad 0\quad 16]$

(E) None of them.

(11)Further to Problem (8), which of the following output feedback controller can stabilize this plant? (2%)

(A) $D(s)=\dfrac{s}{s+1}$, (B) $D(s)=\dfrac{s}{s+10}$, (C) $D(s)=\dfrac{s}{s-1}$, (D) $D(s)=\dfrac{s}{s-10}$

(E) None of them

(12) Further to Problem (5) and assume that

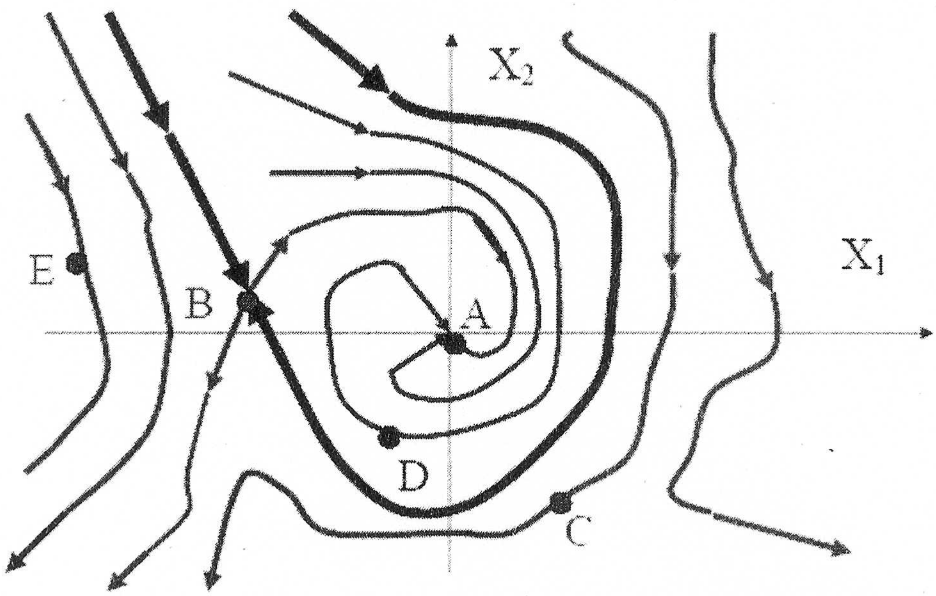
$$F=\begin{bmatrix}0&1&0\\0&0&1\\-6&-11&-6\end{bmatrix},\ G=\begin{bmatrix}4\\5\\1\end{bmatrix},\ H=\begin{bmatrix}1&0&0\end{bmatrix},\ J=0 \qquad \text{and}$$

an observer of a minimal order is required with poles at $s=-2\pm j3.464$. Which of the following observer gain will suffice this need? (2%)

(A) $\tilde{L}=\begin{bmatrix}-2\\17\\1\end{bmatrix}$, (B) $\tilde{L}=\begin{bmatrix}17\\-2\\1\end{bmatrix}$, (C) $\tilde{L}=\begin{bmatrix}-2\\17\end{bmatrix}$, (D) $\tilde{L}=\begin{bmatrix}17\\-2\end{bmatrix}$,

(E) None of them.

(13) Consider the following nonlinear system $\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\f(x_1,\ x_2)\end{bmatrix}$ with the following phase plane



Which states on the above phase plane is stable equilibrium? (2%)

(A) state A, (B) state B, (C) state C, (D) state D, (E) state E.

第二部份：問答與計算題

94 學年度 動力機械工程學 系（所） 乙 組碩士班入學考試

科目 控制系統 科目代碼 1601 共 8 頁第 5 頁 *請在試卷【答案卷】內作答

● 請將以下題目的答案填寫在答案卷上

(14) Please choose from the following statements THREE wrong statements and briefly explain why they are wrong. (3% each)

(A) Given the same plant, an observer-based state feedback control system can never achieve a performance better than an adequately designed output controller.

(B) A stabilizable but undetectable plant can be stabilized by a full state feedback controller.

(C) A first-order hold (FOH) performs significantly better than a zeroth-order hold (ZOH) as a D/A converter. Hence, when the performance is more important than its cost, a FOH should be used instead of a ZOH.

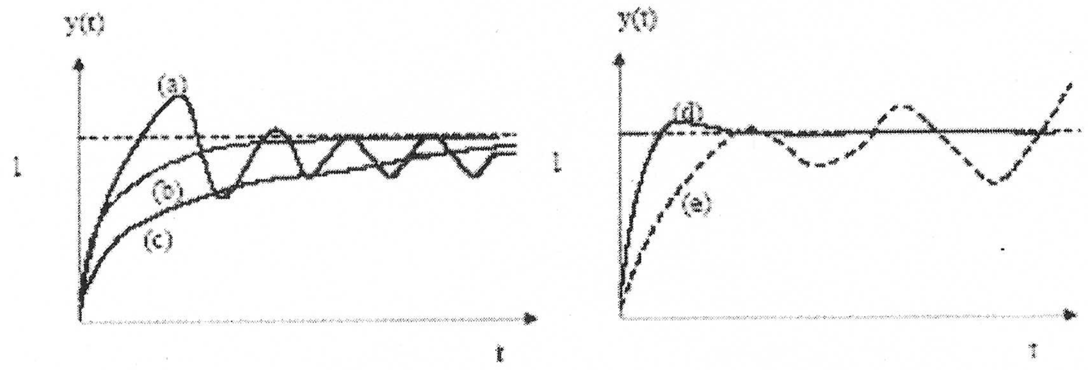
(D) A digital control system always has an overshoot larger than an analog control system no matter what their bandwidths are.

(E) Contrallability and observability can not be ascertained when only the transfer function of a plant is available.

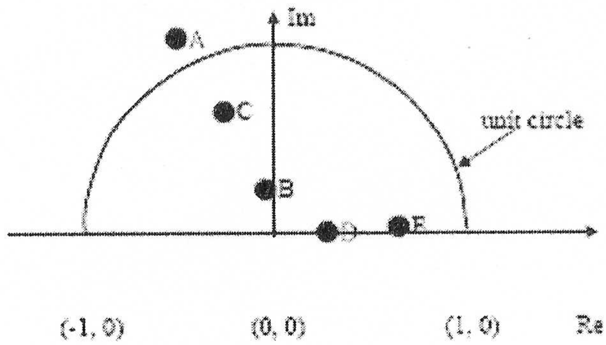
(F) In practice, if a plant is stabilizable using observer-based state feedback, it can also be stabilized by an output feedback controller.

(G) A reduced-order observer is better than a full-dimensional observer in regard of both its cost and accuracy of state estimation.

(15)Consider step responses of discrete systems (a), (b) , (c), (d), (e) shown below

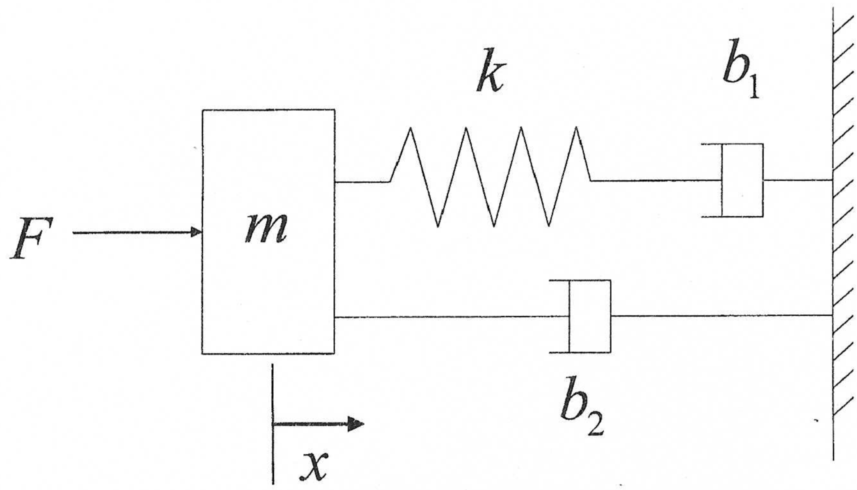


and the pole locations of these discrete systems on the z-plane as shown below



Please match the step responses of systems (a), (b) , (c), (d), and (e) with pole locations (A), (B), (C), (D), (E) on the z-plane. (5%)

(16) Determine the transfer function $\frac{x(s)}{F(s)}$ for the following system. Is it a minimum-phased system? (10%)



F : force input, m : mass, k : spring constant, b_1, b_2 : damping constants

x : displacement of the mass

(17) Find the initial and final values of $y, \dot{y}, \ddot{y}$, for a unit step input for the following transfer functions.

(a) $G(s)=\frac{y(s)}{u(s)}=\frac{2}{s^2+8s+10}$ (3%)

(b) $G(s)=\frac{y(s)}{u(s)}=\frac{s^2+4s+5}{s^2+2s+5}$ (3%)

(18) If the step response of a linear system G is $\frac{t^2}{2}-\frac{1}{3}+\frac{1}{2}e^{-t}-\frac{1}{6}e^{-3t}$, find the output response to the following inputs.

(a) $u(t)=3t$. (4%)

(b) $u(t)=\sin\omega t$ (6%)

(19) A stable system $G(s)$ is said to "undershoot" if it initially starts off in the wrong direction (e.g. in response to a desired step input).

(a) Show that the definition of undershoot is equivalent to saying that the first non-zero initial derivative of the output has an opposite sign to that of its steady-state value. (3%)

(b) Show that the order of that derivative, r, turns out to be the relative degree of $G(s)$, i.e. the difference between the order of the denominator and the numerator. (3%)

(c) Prove that a system "undershoots" if and only if it has an odd number of real right-half plane zeroes. (4%)

Hint: We can assume, without loss of generality, that the D.C. gain of the system is one. Thus, start your proof by setting the stable system as

$$G(s)=\frac{\left(1-\frac{s}{z_1}\right)\left(1-\frac{s}{z_2}\right)\cdots\left(1-\frac{s}{z_n}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\cdots\cdots\left(1-\frac{s}{p_{n+r}}\right)}$$

(20) Given a plant $G(s)=\frac{1}{s^2-1}$

(a) plot its frequency responses (including the magnitude plot and the phase plot), (5%)

(b) design a PD controller so that the closed-loop system, when under a step input, has zero overshoot and a rise time of $1\sec$. Plot the associated root-locus. (5%)

(21)A magnetic-tape-drive speed control system is shown in the following figure. The speed sensor is slow enough that its dynamics must be included. The speed measurement time constant is $\tau_m = 0.5\sec$, the reel time constant $\tau_r = \dfrac{J}{b} = 4\sec$, the output shaft damping constant is $b = 1 N \cdot m \cdot s$, and the motor time constant is $\tau_1 = 1\sec$.

(a) Determine the necessary gain K if the steady-state error is required to be less than 7% of the reference speed setting. (4%)

(b) With the gain determined from (1), investigate the stability of the system. (4%)

(c) Estimate the gain and phase margins of the system. Is this a good system design? Why? (6%) (Hint: For the phase margin, you may want to use the bisection method to roughly estimate the real root of a 3rd degree polynomial.)