94 學年度 動力机械工程 系（所）甲,乙,丙,丁 組碩士班入學考試

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1. Solve the following equations:

(i) $\frac{d^2y}{dt^2} - 3y^2 = 0$, $y(0) = 2$; $\frac{dy(0)}{dt} = 4$ (10%)

(ii) $\frac{dy}{dx} = \frac{y - 4x}{x - y}$ (10%)

2. Use Laplace transforms to solve the following equations for y(t)

$\frac{d^2y}{dt^2} + 4y = f(t)$,

where f(t)=1 for 0< t < 1 and f(t)=0 everywhere else. The initial condition for y are $y(0) = 0$ and $\frac{dy(0)}{dt} = 0$ (15%)

3. If a vector field **F** is given by $\mathbf{F} = y^2\mathbf{i} + x^2\mathbf{j} + z^2\mathbf{k}$, evaluate $\iiint_V \nabla \cdot \mathbf{F} dV$ where V is the upper half ($z \geq 0$) of the volume within the sphere $x^2 + y^2 + z^2 = 1$. (10%)

4. Find a two-by-two matrix with the eigenvalues 2 and 6, and the corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. (10%)

5. Find the Fourier coefficients of the periodic function f(x).

$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$ (10%)

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6. Use separation of variable to find the temperature T(x,t) in a laterally insulated copper bar 0.8 m long if the initial temperature is $100\sin(\pi x/0.8)\ ^{o}C$ and the ends are kept at $0\ ^{o}C$. How long will it take for the maximum temperature in the bar to drop to $50\ ^{o}C$? Physical data for the copper: density $8920 kg/m^3$, specific heat $92\ cal/kg^{o}C$, thermal conductivity $95\ cal/m$ sec ^{o}C. (15%)

7. Find all solution for z, that $e^z = 1 + i$ (10%)

8. A potential function **u** is generally a solution of Laplace's equation. **u** =constant are called equipotential surface, while **v** = constant generally represent direction of force, where **v** is a conjugate harmonic function of **u**. **u** and **v** satisfy the Cauchy-Riemann Equation ($\mathbf{u}_x = \mathbf{v}_y$ and $\mathbf{u}_y = -\mathbf{v}_x$). let $\mathbf{u} = x^2 - y^2$, find **v**. (10%)