國 立 清 華 大 學 命 題 紙

九十三學年度 動力機械工程學 系 甲、乙、丙、丁 組碩士班研究生招生考試

科目 工程數學 科號 1503, 1603, 1703, 1803 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following ordinary differential equations.

   (a) Find all initial conditions such that the initial value problem

   $$\left(x^2+2x\right)y' = 2\left(x+1\right)y, \quad y\left(x_0\right) = y_0$$

   has (i) no solution, (ii) more than one solutions, and (iii) precisely one solution. (6%)

   (b) $\left(4x^2+4x+1\right)y'' - \left(4x+2\right)y' - 12y = 3x/\left[x+\left(1/2\right)\right]$. (14%)

2. Solve the difference equation

   $$3y(t) - 4y(t-1) + 2y(t-2) = t,$$

   using the Laplace transform if y(t) = 0 for t < 0. (10%)

3. Find the work done in moving a particle once around a circle C in the xy plane, as shown in Figure 1, if the circle has center at the origin and radius 3 and if the force field is given by

   $$\mathbf{F} = \left(2x-y+z\right)\mathbf{i} + (x+y-z^2)\mathbf{j} + (3x-2y+4z)\mathbf{k}, \quad (10\%)$$

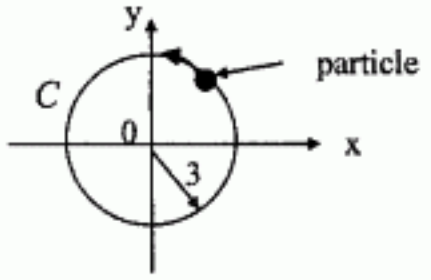

Figure 1

4. Consider the system governed by the following equation:

   $$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}\left(t\right) \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

   (a) Find the eigenvalues and eigenvectors of matrix **A**. (6%)

   (b) If the initial values are $x_1(0) = x_2(0) = 1000$, determine the solution of **x**(t). (8%)

   (c) Is this a stable system? (2%)

5. Compute the following values where $i=\sqrt{-1}$:

   (a) $(1+i)^{20}$. (3%)

   (b) $(\cos\theta + i\sin\theta)^6$ where $\theta = 30^o$. (3%)

   (c) $\log(1+i)$. (4%)

   (d) i^i. (4%)

6. A period function $f(x)=f(x+2\pi)$ can be represented by a trigonometric series

$$f(x)=a_0+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

   Please find the coefficients a_0, a_n, and b_n. (10%)

7. Consider a one-dimensional wave equation,

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$

   where $u(x, t)$ is the deflection of the string and the length of string is l. The string is fixed at the ends $x = 0$ and $x = l$ at all time. Please obtain the solution with the following initial conditions. (Please use the separation of variable approach.)

$$u(x,0)=\begin{cases}\dfrac{2}{l}x & when\ \ 0<x<\dfrac{l}{2},\\ \dfrac{2}{l}(l-x) & when\ \ \dfrac{l}{2}<x<l.\end{cases}$$

   and

$$\left.\frac{\partial u}{\partial t}\right|_{t=0}=0.$$ (20%)