國立清華大學命題紙

九十二學年度 動力機械工程學 系（所） 乙 組碩士班研究生招生考試

科目 控制系統 科號 1401 共 5 頁第 1 頁 *請在試卷【答案卷】內作答

Note: Assuming whatever variables and parameters if necessary

Prob. 1. Consider the following magnitude plot as shown in Figure 1

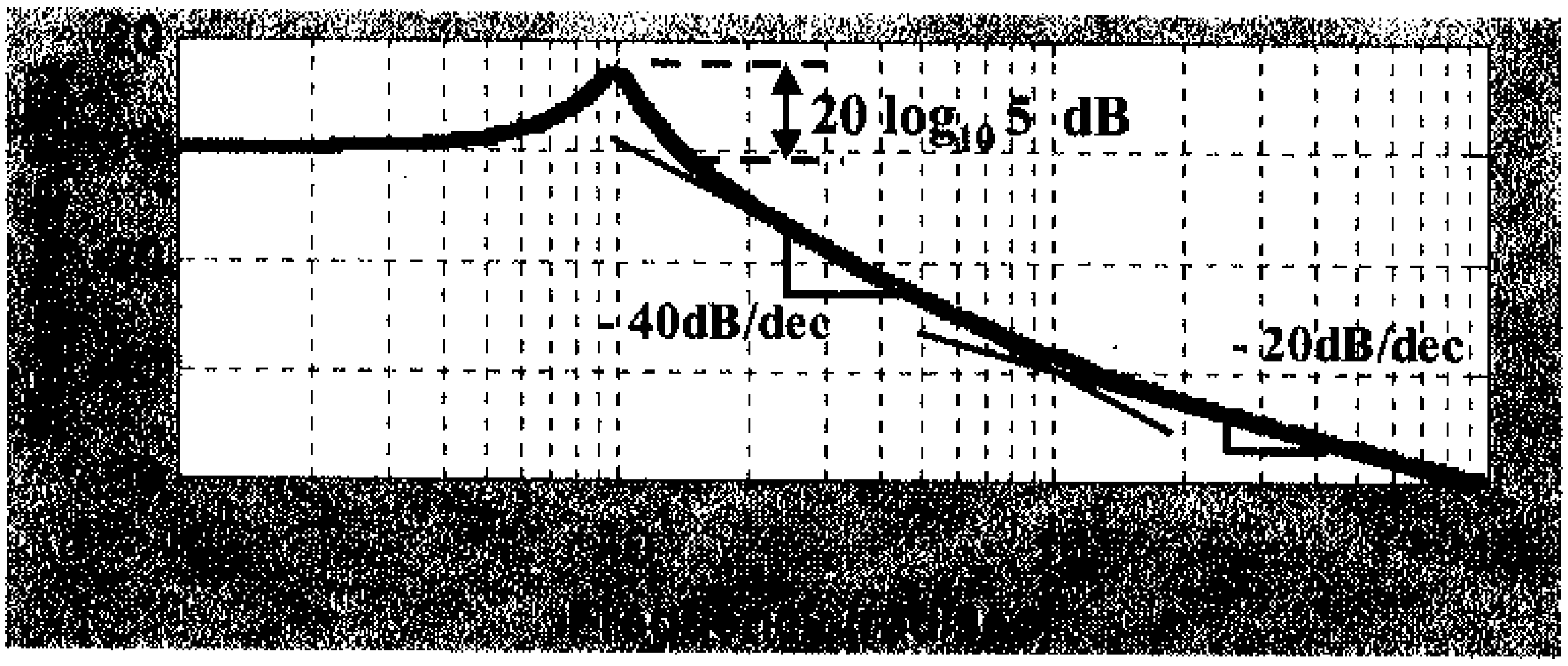

Figure 1

(a) List all of the transfer functions, Gp, that could give the following frequency response. (6%) (Hint: there are four possible transfer functions.)

(b) Which of the possible Gp are open-loop stable? (2%)

(c) Which of the possible Gp can be stabilized with unity feedback and a proportional controller? (6%)

Prob. 2 Consider the following block diagram as shown in Figure 2

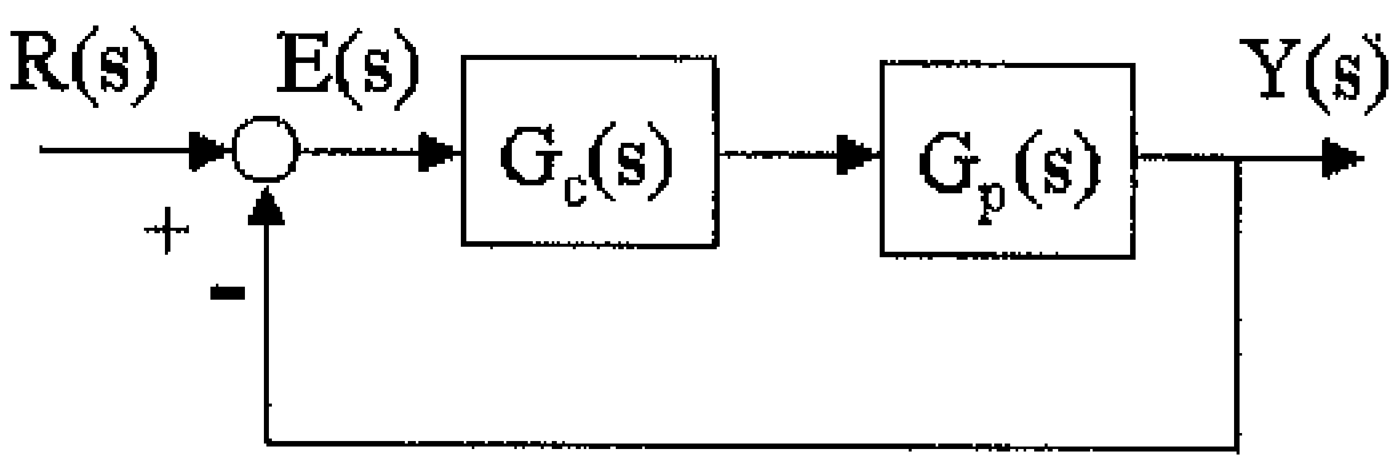

Figure 2

where $G_p(s) = \dfrac{1}{(s+2)(s+10)}$. and $G_c(s) = \dfrac{K}{s}$, $K = 100$

(a) For the Nyquist plot of $G_c(s)G_p(s)$, what is the real axis crossing if there is? (6%)

(b) Draw the Nyquist plot of $G_c(s)G_p(s)$. (4%)

(c) Use Nyquist Criterion to figure out the stability of the closed-loop system. (2%)

(d) Use Nyquist to find the Gain Margin of the system. (2%)

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科目 控制系統 科號 1401 共 5 頁第 2 頁 *請在試卷【答案卷】內作答

Prob. 3 Consider the Nichols plots of an **uncompensated** and a **compensated** system are shown in Figure 3.

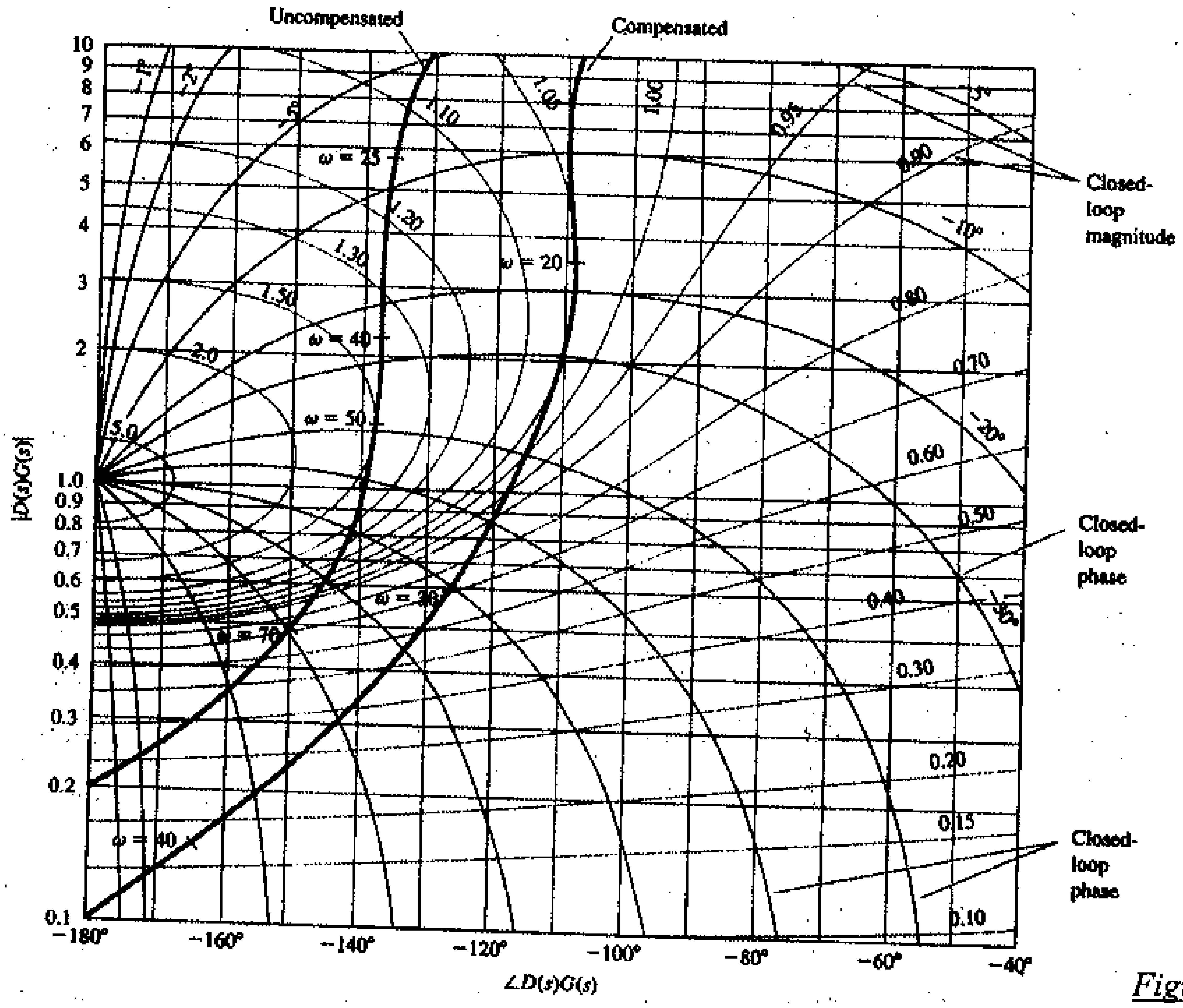

Figure 3

(a) What are the resonance peaks of **each system**? (2%)

(b) What are the phase and gain margins of **each system**? (4%)

(c) What are the bandwidths of **each system**? (2%)

(d) What type of compensation is used? (Lead or Lag) (2%)

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Prob. 4 The signal $f(t) = \operatorname{sinc}^2(5 \cdot \pi \cdot t)$ vs. time, t, is shown in Figure 4. The sampling rate is ω Hz. The corresponding sample signal is $\bar{f}(t)$.

What is the minimum sampling rate $\omega_{\min}$ required to recover $f(t)$ from its samples $\bar{f}(t)$ without signal aliasing or folding. Please briefly explain your statement. (2 %)

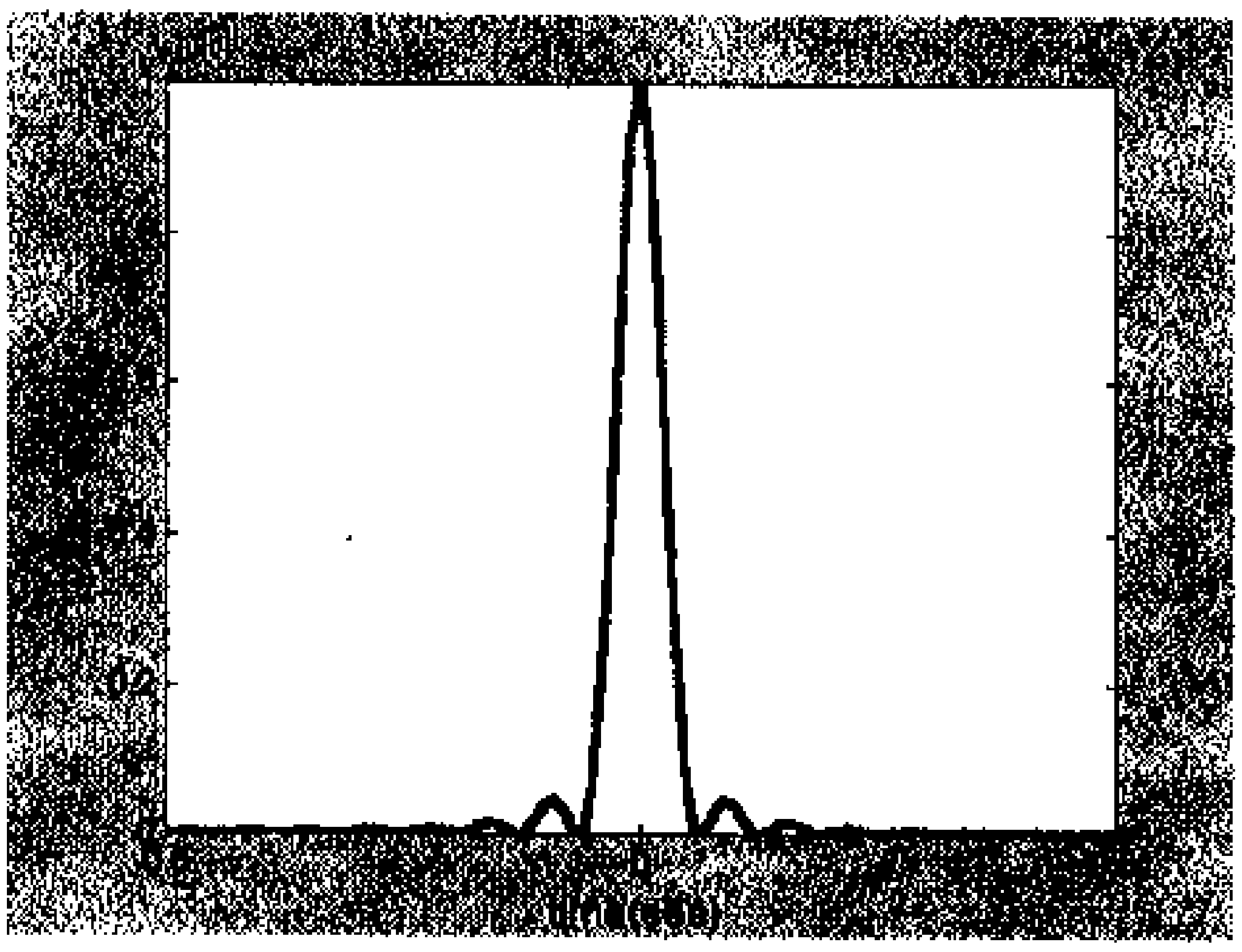

Figure 4

Prob. 5 Consider a state-space representation of a system described as

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u \equiv AX + Bu$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} X$$

(a) Indicate the controllabllitiy and observability of each state, respectively. (6 %)

(b) Obtain a control canonical form for the above system. (2 %)

(c) Obtain an observer canonical form for the above system. (2 %)

Prob.6 Consider a state-space representation of a system described as

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \equiv AX + Bu$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} X \equiv CX$$

(a) If there exists a non-singular transformation matrix T, and $X = TZ$, such that the above system can be transformed to the control canonical form as shown below

$$\frac{d}{dt}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \equiv FZ + Gu$$

Find T. (6 %)

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(b) Find a state feedback control law: $u=-KX$, such that the closed-loop poles are located at $-4+j4$ and $-4-j4$. (7 %)

(c) Find the gain vector L for a full-state estimator,

$$\dot{\hat{X}} = A\hat{X} + Bu + L(y - C\hat{X})$$

such that the dynamics of the error-estimator has repeated characteristic roots of -8. (7 %)

Prob. 7 Figure 7 shows a wheeled-inverted-pendulum system. In the system, a link is attached to a wheel which can roll freely on the horizontal surface. A motor is installed at the joint where the link and the wheel are connected. The motor is for providing a torque T to control the motion of the link. However, when T is exerted, its reactive torque also acts on the wheel to make it roll.

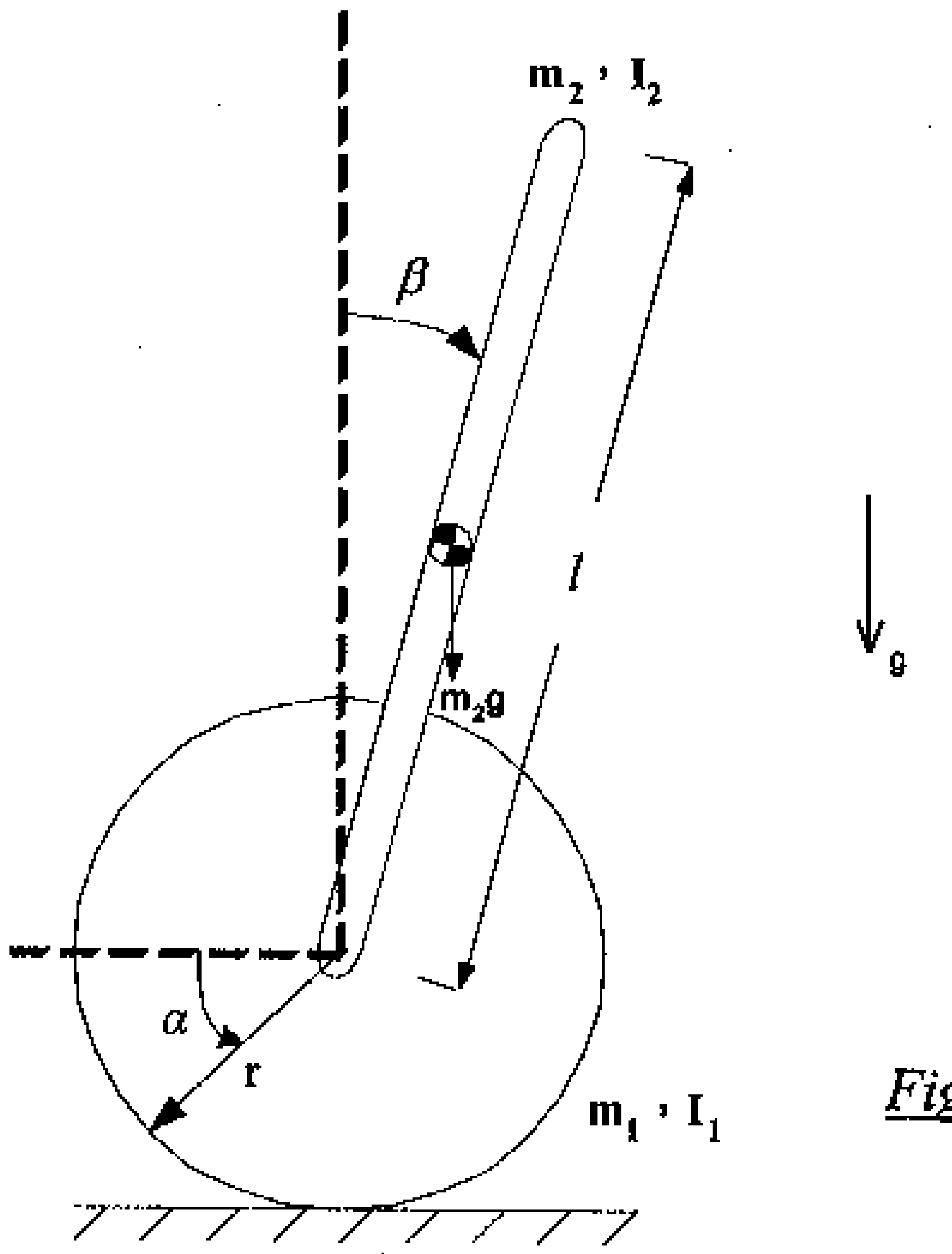

Figure 7

Using the Lagrange's method, the dynamic equations of the system can be derived as:

$$(m_1r^2 + I_1 + m_2r^2)\ddot{\alpha} - m_2\ell r\cdot\cos(\beta)\cdot\ddot{\beta} + m_2\ell r\cdot\sin(\beta)\cdot\dot{\beta}^2 = T$$
$$(m_2\ell^2 + I_2)\ddot{\beta} - m_2\ell r\cdot\cos(\beta)\cdot\ddot{\alpha} - m_2 g\ell\sin(\beta) = T$$

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in which α is the absolute rotation angle of the wheel, β is the angle of the link deviated from the vertical line, $m_1(m_2)$ and $I_1(I_2)$ are respectively the mass and the inertia of the wheel (link), r is the radius of the wheel, ℓ is the length of the link and g is the gravity.

(a) Defining $\begin{bmatrix}\alpha & \beta & \dot{\alpha} & \dot{\beta}\end{bmatrix}^T$ as the state vector, write down the state equations and verify that $\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T$ is one of the equilibrium points. Also find the other equilibrium point(s). (6%)

(b) Examine the system stability by linearizing the system around $\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^T$. What are the open-loop poles? (5%) Is the system controllable? (4%)

Prob. 8 (a) Show that the systems in Fig. 8(a) and Fig. 8(b) are analogous systems. (Show that the transfer functions of the two systems are of identical form). (8%)

(b) Assuming that $k_1(C_1), k_2(C_2), b_2(R_2) > 0$, draw the root locus of the systems as $b_1(R_1)$ varies from zero to infinity. (7%) (Notice that b_1, b_2 are linear dampers)

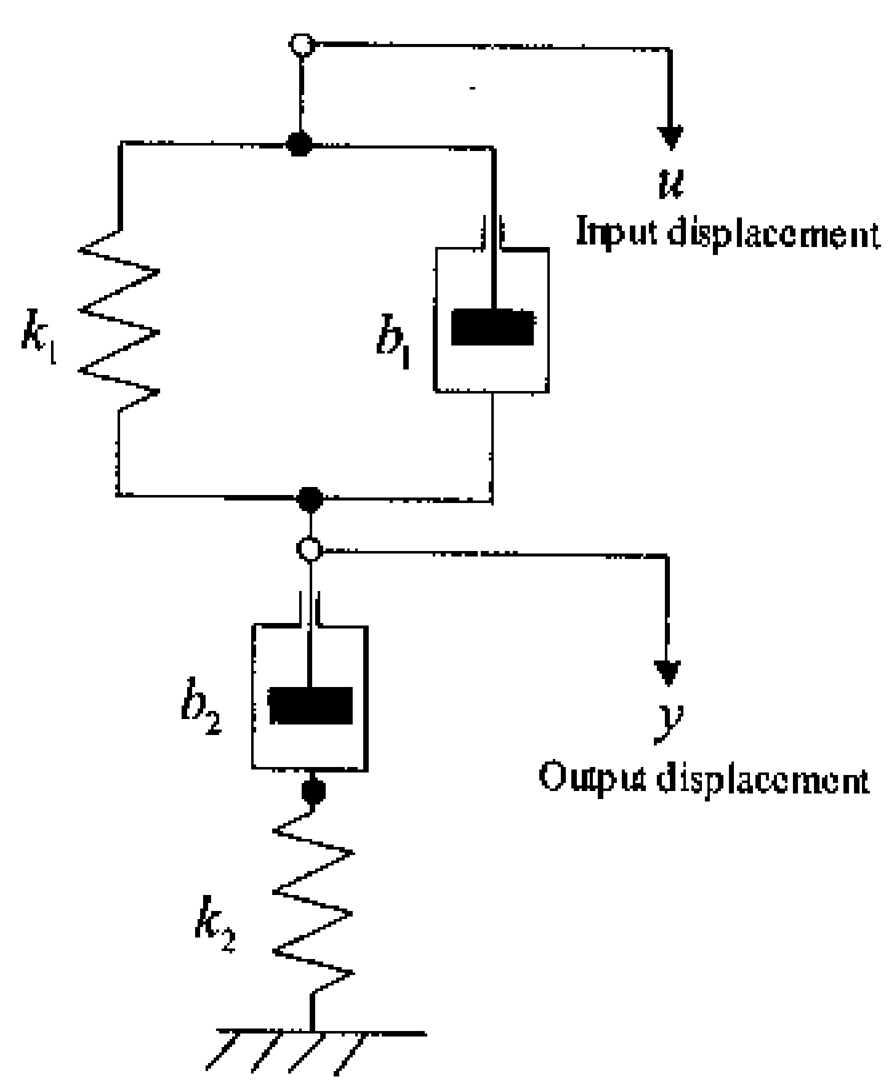

Figure 8(a)

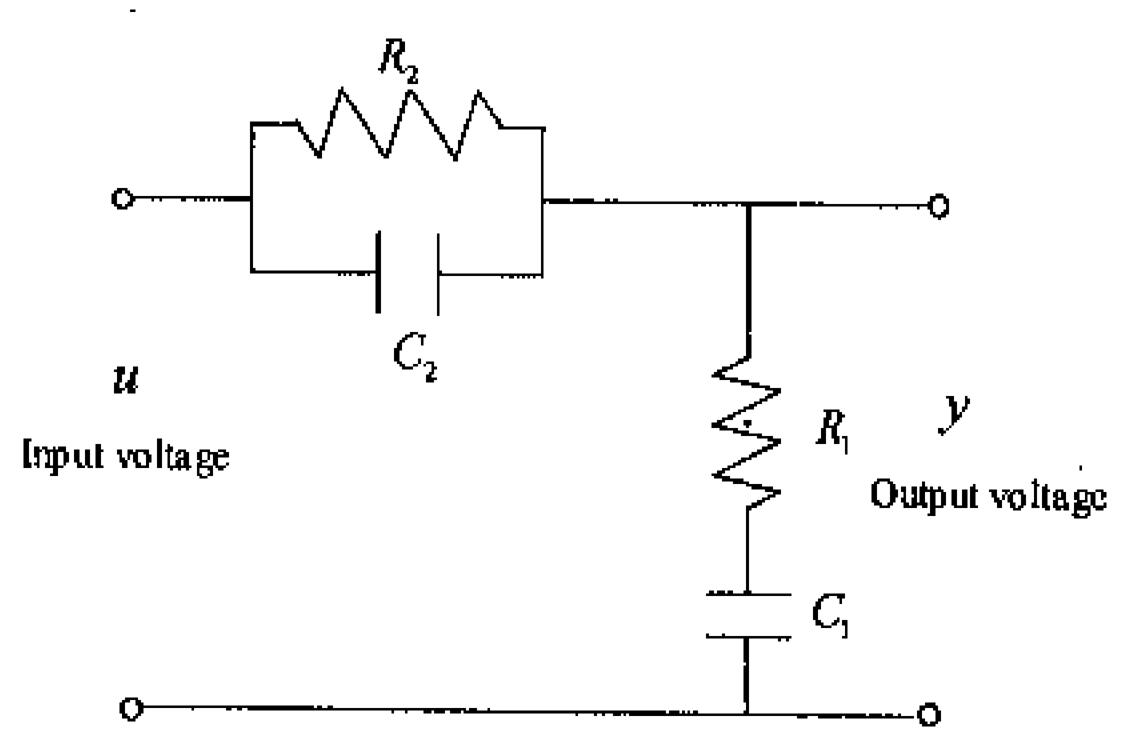

Figure 8(b)

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