國 立 清 華 大 學 命 題 紙

九十二學年度 動力機械工程學 系 甲、乙、丙、丁 組碩士班研究生招生考試
科目 工程數學 科號 1303, 1403, 1503, 1603 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Answer **TRUE** or **FALSE** for the following questions with brief but reasonable explanation.
   (a) Let A be an n by n matrix with m distinct eigenvalues, $\mu_1, \cdots, \mu_m$, m < n,. then det A = $\mu_1 \mu_2 \cdots \mu_m$. (3%)
   (b) If $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are two solutions of $\boldsymbol{Ax = b}$, then $\boldsymbol{x_1 - x_2}$ is in $\boldsymbol{N(A)}$, where $\boldsymbol{A}$ is an n by n matrix, $\boldsymbol{x}$ and $\boldsymbol{b}$ are n by 1 vectors, and $\boldsymbol{N(A)}$ denotes the nulls space of $\boldsymbol{A}$. (3%)
   (c) $Ae^{At} = e^{At}A$. A is an n by an matrix. (3%)
   (d) Legendre's equation, $(1-x^2)y''(x) - 2xy'(x) + n(n+1)y(x) = 0$, is a Sturm-Liouvill equation, $[r(x)y'(x)]' + [q(x) + \lambda p(x)]y(x) = 0$, by properly choosing r(x), q(x), and p(x). (3%)
   (e) $\dot{x} = -sign\ x(t),\ x(0) = 0$ where

   $$-sign\ x(t) = \begin{cases} 1 & \text{if } x \ge 0 \\ -1 & \text{if } x<0 \end{cases}$$

   The differential equation has continuously differentiable solution x(t) for $t \ge 0$. (3%)

2. Consider the differential equation
   $\ddot{y}(t) + 2\dot{y} + 2y = f(t)$
   (a) Let $f(t) = e^{-t}\sin t + \cos 2t$. Solve the above differential equation. (10%)
   (b) Let $f(t)$ be described as shown in Figure 1. Solve the above differential equation. (10%)

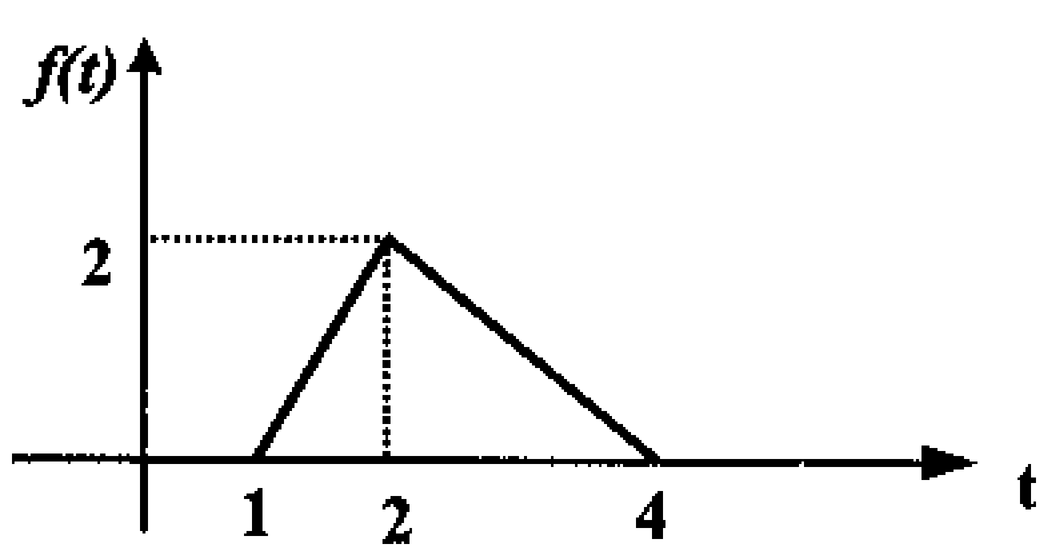

Figure 1

3. By changing variables from (x, y) to (u, v) according to the transformation $x = x(u, v)$, $y = y(u, v)$, show that the area A of a region R bounded by a simple closed curve C is given by

$$A = \iint_R \left| J\left(\frac{x,y}{u,v}\right) \right| dudv \quad \text{where} \quad J\left(\frac{x,y}{u,v}\right) \equiv \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

is the Jacobian of x and y with respect to u and v. **Hint:** $A = \frac{1}{2}\int(xdy - ydx)$ and use Green's theorem. (10%)

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4. Solve the partial differential equation of one-dimensional heat flow

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

with the boundaries $u_x(0,\ t)=0,\ u_x(\pi,\ t)=0,\ u(x,\ 0)=\sin x$. (20%)

5. Prove the following integral (10%)

$$\int_0^\infty \frac{\cos\omega x\ \sin\omega}{\omega}\,d\omega=\begin{cases}\pi/2 & if \quad 0\le x<1\\ \pi/4 & if \quad x=1\\ 0 & if \quad x>1\end{cases}$$

6. Show

(a) $\int_{-\infty}^{\infty}\frac{\cos sx}{k^2+x^2}\,dx=\frac{\pi}{k}e^{-ks}$ (b) $\int_{-\infty}^{\infty}\frac{\sin sx}{k^2+x^2}\,dx=0$ $(s>0,\quad k>0)$

by the residue integration method. (10%)

7. Integrate the following function counterclockwise over the given path C (15%)

$$f(z)=\frac{z}{|z|^2}\qquad C:|z|=1$$