

九十學年度 動力機械系甲、乙、丙、丁組碩士班研究生招生考試
 科目 工程數學 科號 1303, 1403, 1503, 1603 共 2 頁第 1 頁

*請在試卷【答案卷】內作答

1. Solve the Sturm-Liouville problem

$$y''(x) + \lambda y(x) = 0, \quad y(-\pi) = y(\pi), \quad y'(-\pi) = y'(\pi) \quad (15\%)$$

where λ is a real value.

2. Solve the simultaneous differential equations

$$\begin{aligned} \dot{x}(t) &= x - y + e^{-t} \\ \dot{y}(t) &= 2x - 2y + \sin(2t)e^{-t} \end{aligned} \quad (15\%)$$

3. (a) Find the inverse Laplace Transform for

$$F(s) = \frac{1}{s(1 - e^{-s})} \quad (5\%)$$

- (b) If the Laplace transform of $f(t)$ is

$$F(s) = \frac{s}{(s^2 + a^2)}$$

where a is a constant. Find $f(t)$ for $t \rightarrow \infty$. (5%)

4. Given Points $A(1,0,2)$, $B(4,5,0)$, $C(0,-3,5)$, $D(-2,0,7)$, and $E(0,-1,7)$ in the space, please find:

- (a) the normal unit vector of plane **CDE**. (5%)
 (b) the equation, which is the perpendicular bisector of line **AB**. (5%)
 (c) the coordinate of piercing point **M**, where the line **AB** intersects the plane **CDE**. (5%)

5. (a) Find the Fourier series of periodic function as follows:

$$f(x) = \begin{cases} k & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 < x < 3\pi/2 \end{cases} \quad (10\%)$$

- (b) Show $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ (5%)

九十學年度 動力機械 系甲、乙、丙、丁 組碩士班研究生招生考試
 科目 工程數學 科號 1303, 1403, 1503, 1603 共 2 頁第 2 頁

*請在試卷【答案卷】內作答

6. Please solve the following partial differential equation

$$\frac{\partial y}{\partial t} = a \frac{\partial^2 y}{\partial x^2}$$

subject to the initial and boundary conditions

$$y(x, 0) = y_0, \quad y(0, t) = 0 \quad \text{and} \quad y(\infty, t) = y_0.$$

$$\text{(Note: } \int_0^{\infty} e^{-a\lambda^2 t} \cos \lambda x \, d\lambda = \frac{1}{2} \left(\frac{\pi}{at} \right)^{1/2} e^{-x^2/4at} \text{)} \quad (10\%)$$

7. For a complex number z , $z = x + iy$

(a) Is $|\cos(z)| \leq 1$? Why? (7%)

(b) Evaluate $\oint_C \frac{1}{z} dz$, where C is any simple closed contour in the z -plane.

(13%)