

QUESTION 1 (18%)

Solve the following ordinary differential equations:

(a)
$$\frac{d^3 y}{dx^3} - 4 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + 6y = (e^{-2x} - 2)^2 \quad (10\%)$$

(b)
$$y \frac{d^2 y}{dx^2} = \left(\frac{dy}{dx} \right)^2 - \frac{dy}{dx} \quad (8\%)$$

QUESTION 2 (7%)Find the inverse Laplace Transform for $F(s) = \ln\left(1 + \frac{1}{s^2}\right)$ **QUESTION 3 (15%)**Consider a 3×3 matrix $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$ (a) Find the eigenvalues and the correspond eigenvectors of matrix A . (10%)(b) Find an orthogonal matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (5%)**QUESTION 4 (10%)**Find the distance between the two straight lines: $(x, y, z) = (3 + t, 1 - 2t, 2 + 2t)$ and $(x, y, z) = (7 + t, 1 - 2t, -3 + 2t)$, where t is a parameter.**QUESTION 5 (25%)**(a) Let $f(x)$ be a periodic function with period 2π . If $f(x)$ is piecewise continuous in the interval $-\pi \leq x \leq \pi$, then $f(x)$ can be represented by a Fourier series of the form

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Derive the Euler formulas for the coefficients a_0 , a_n and b_n . (15%)(b) What conditions are needed, respectively, for $a_0 = 0$, $a_n = 0$, and $b_n = 0$? (10%)

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科目：工程數學 科號：1303 1403 1503 1603 共2頁第2頁 *請在試卷【答案卷】內作答

QUESTION 6 (25%)

(a) Integrate the complex function $f(z)$ counterclockwise around the circle $|z|=3$ by using the Cauchy's integral formula. (10%)

$$f(z) = \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)}$$

(b) Evaluate the following integral by using the residue theorem. (15%)

$$\int_{-\infty}^{\infty} \frac{x \cos x}{x^2 - 2x + 10} dx$$