八十八學年度 動力機械 系（所）甲、乙、丙、丁組碩士班研究生招生考試

科目 工 程 數 學 科號 1203 1303 1403 1503 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

QUESTION 1

Find the general solution for the following system of ordinary differential equations

$$\begin{cases} \ddot{x} + 3x + y = \sin^2 t \\ \ddot{y} + 2y + 2x = \cos^2 t \end{cases}$$

where $\ddot{x}$ and $\ddot{y}$ denote, respectively, the second derivatives of x and y with respect to t.

(15%)

QUESTION 2

Consider the Laplace transformation $L[f(t)] = F(s)$.

(a) Prove the theorem $L[\frac{f(t)}{t}] = \int_s^\infty F(u)\,du$ (5%)

(b) Use the theorem in part (a) to show that $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$. (5%)

QUESTION 3

(a) Show that the vectors $\begin{Bmatrix}1\\1\\0\end{Bmatrix}$, $\begin{Bmatrix}0\\1\\1\end{Bmatrix}$, $\begin{Bmatrix}1\\0\\1\end{Bmatrix}$ form a basis for the vector space $\Re^3$. (10%)

(b) Use the basis given in part (a) to construct an orthonormal basis for the same vector space $\Re^3$ with the Gram-Schmidt orthonormalization process. (15%)

QUESTION 4

Solve the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

with the following initial conditions and boundary conditions

$$u(x,0) = f(x), \quad \partial u(x,0)/\partial t = g(x)$$

$$u(0,t) = 0, \quad u(L,t) = a\sin(\omega t)$$

where c, a and ω are all constants. (25%)

QUESTION 5

Use the residue theorem to evaluate

(a) $\oint_C \frac{5z^2 - 3z + 2}{(z-1)^3} dz$

where C is an arbitrary simple closed curve enclosing the point z =1. (10%)

(b) $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$ (15%)