1. Figure 1 shows a single-link robot driven by a DC servomotor. When a control voltage u is applied to the motor armature, due to the electro-mechanical interaction, the torque developed (τ_m) is directly proportional to the armature current (i) so that

$$\tau_m = K_t i ,$$

where K_t is a motor-torque constant. To amplify the torque, a gear transmission with gear ratio n is used between the motor shaft and the robot link. Let I_m be the moment of inertia of the motor shaft and Gear 1 combination, and I_L be the moment of inertia of the robot link together with Gear 2. A rotational damper of coefficient b is place at the end of Gear 1 to model the possible viscous-friction.

a) What is the relation between the motor speed ω_m and the link speed $\dot{\theta}$? (5%)

b) Based on the result in part (a), determine the equivalent moment of inertia referred to the motor shaft. (5%)

c) Assuming that the armature inductance L is negligible, show how the torque τ_m is related to the control voltage u and motor speed ω_m. (5%)

d) Using u as input and θ as output, obtain a differential equation describing the dynamics of the system. (5%)

e) Using the result in part (d), find the condition when the inductance L can be assumed negligible. (5%)

f) Assuming I_m and I_L are known, determine the optimal gear ratio n so that the acceleration capability of the system is maximized. (5%)

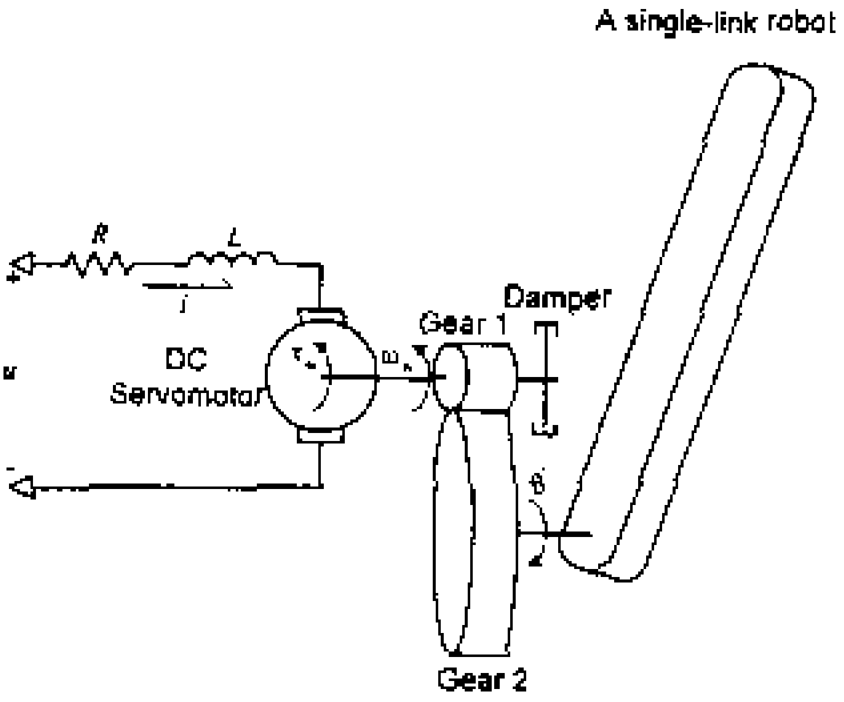


Figure 1. A single-link robot driven by a DC servomotor

2. When integral control is used in engineering systems, a phenomenon known as *integrator windup* may occur when the closed-loop systems encounter large commands or exceptional disturbances.
   a) Explain how *integrator windup* occurs and how it affects the performance and stability of the control system. (5%)
   b) Recommend a remedy for this phenomenon. (5%)

3. Consider a system with an unstable plant as shown in Figure 2.

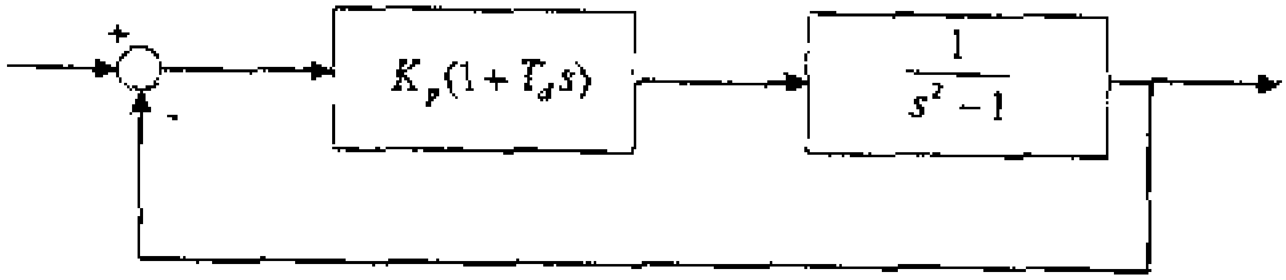


Figure 2. PD control of an unstable plant

   a) Design a proportional-plus-derivative controller (that is, determine the values of K_p and T_d) such that the damping ratio ζ of the closed-loop system is 0.5 and the undamped natural frequency ω_n is 1 rad/sec. (4%)
   b) What are the gain margin and the phase margin of the control system you design in part (a)? (6%)

4. Give a system with transfer function as

$$g(s) = \frac{4s^2+25s+38}{s^3+9s^2+26s+24}$$

If the state-space representation of this system is

$$\dot{X} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{bmatrix} X + Gu$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X$$

find a, b, and G. (10 %)

5. Consider a system having state-space representation as

$$\dot{X} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u \triangleq F X + G u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} X \triangleq H X$$

(a) Test the stability for the system. (5 %)

(b) Show that there exists state that can not be estimated by observing the input and the output. (5 %)

(c) Show that the poles of the system can be assigned arbitrarily through state feedback control. (5 %)

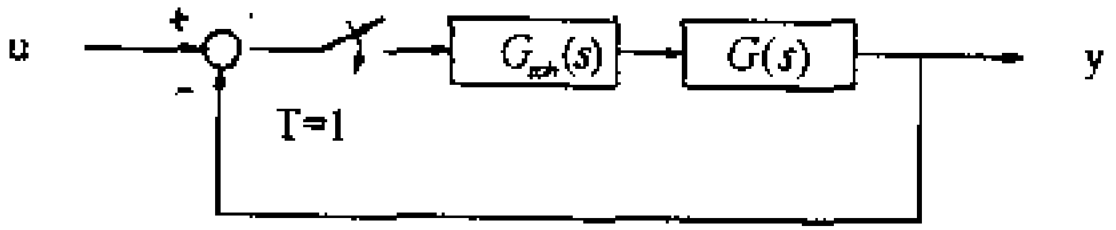
(d) Find a matrix T so that after the linear transformation $X = T Z$, the system can be transformed into the control canonical form as

$$\dot{Z} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} Z + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \triangleq A Z + B u$$

where a_1, a_2 and a_3 are coefficients of the characteristic polynomial of the system. (10%)

(e) Find the state feedback control law $u = K X + v$ to move the poles to -1, -2 and -3. (5 %)

6. Consider the discrete system shown below.



Where $G_{zoh}(s)$ is ZOH and $G(s) = \dfrac{1}{s+2}$

(a) Find $G(z) \triangleq \dfrac{y(z)}{u(z)}$ (5 %)

(b) Test the stability of the system. (5 %)

Note : Z-transform of $G_{zoh}(s)G(s) = (1-z^{-1}) \times$ (Z-transform of $\dfrac{G(s)}{s}$)

Z-transform of $\dfrac{1}{s} = \dfrac{z}{z-1}$

Z-transform of $\dfrac{1}{s+a} = \dfrac{z}{z-e^{-aT}}$

$e^{-2} = 0.135$