國立清華大學命題紙

八十五學年度 動力機械工程學 系（所） 乙 組碩士班研究生入學考試

科目 控制系統 科號 2601 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. A rotational system is shown in Fig. 1. Derive the differential equation for the motion in which θ_o is the output and θ_i is the input. Note that c, k, and I are the damping coefficient, spring constant, and inertia of the mass, respectively. Also notice that n_1 and n_2 are the tooth number of gears whose inertia are negligible. (10%)

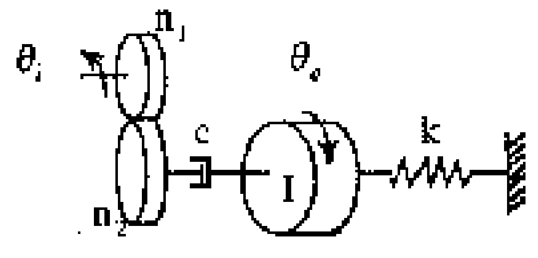

Fig. 1

2. A feedback control system is shown in Fig. 2 (a). The open-loop transfer function is

$$G(s) = \frac{(s+1)}{s(s+3)(s+5)}$$

(a) With K = 150 and the sensor gain $\alpha = 1$, determine the steady-state error of the closed-loop system for the periodic input shown in Fig. 2 (b). (7%)
(b) Find the sensitivity of the closed-loop system with the sensor gain α and discuss the effect of α to the system. (8%)

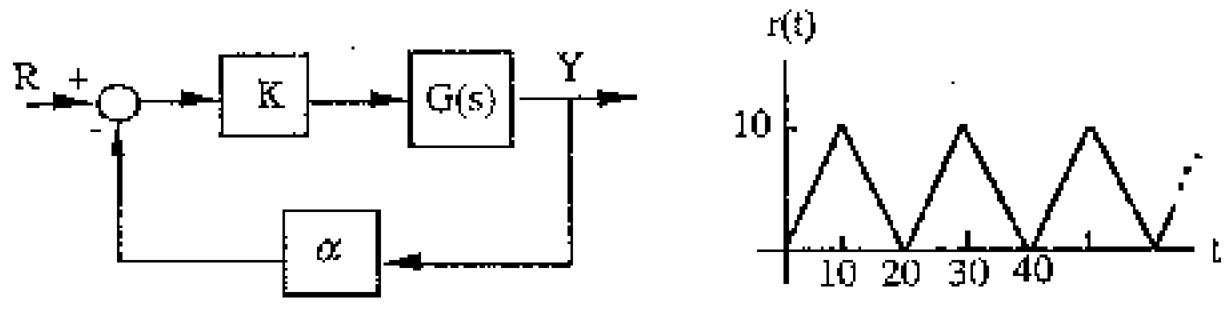

Fig. 2 (a) Fig. 2 (b)

3. The Nyquist plot for the open-loop system KG(s) is given in Fig. 3 where gain K = 2 Based on this plot, determine the range of values of K for which the system will be stable using Nyquist Stability criterion. (10%)

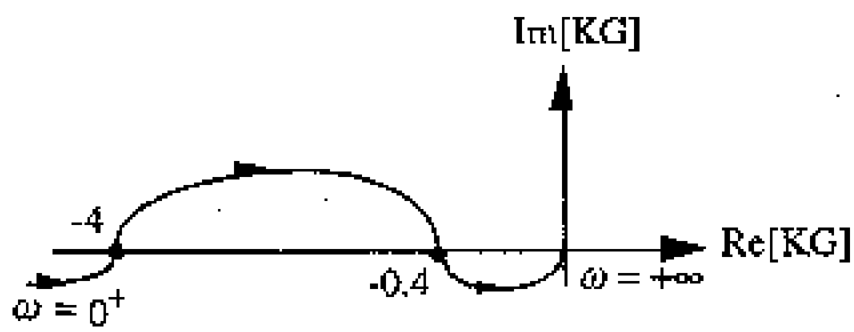

Fig. 3

4. A dc motor shown in Fig. 4(a) is tested in open loop. When unloaded, its velocity response to a step voltage of 5 V is

$$\omega(t) = 100(1 - e^{-10t}) \quad \text{rad/s}$$

When the motor is loaded by T = 6 N-m, its steady-state velocity drops to 70 rad/s to a step voltage of 5 V. The torque-velocity characteristic curve (assuming a straight line) of 5 V is shown in Fig. 4 (b).

(a) Derive the equation of motion for the motor. Note that the disturbance (load) MUST be included. (7%)

(b) The motor is controlled using a PI controller as shown in Fig. 4(c). The tachometer gain is 0.5 V/rad. Determine the proportional gain k_p and integral gain k_I such that the system has damping ratio ζ =0.707 and natural frequency ω_n = 10 rad/s. (8%)

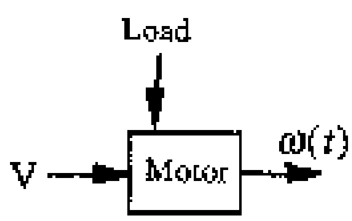

Fig. 4 (a)

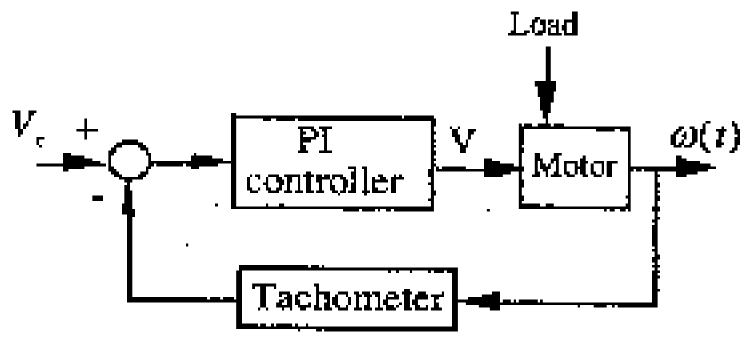

Fig. 4 (c)

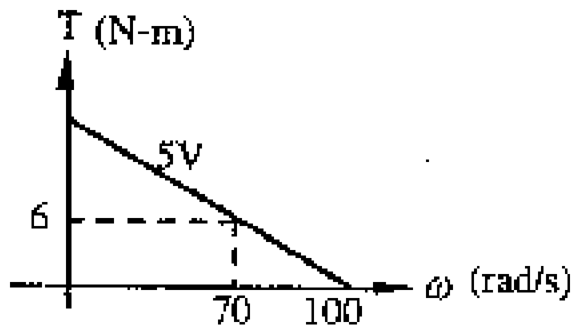

Fig. 4 (b)

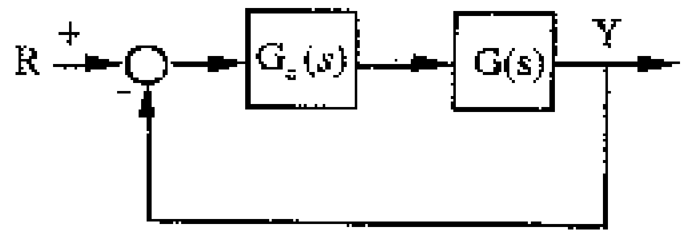

Fig. 5

5. A compensator is given as

$$D(s) = \frac{Ts+1}{\alpha Ts+1}, \qquad 0 < \alpha < 1, \qquad T > 0.$$

(a) Draw the Bode diagram for this compensator. (5%)

(b) Find the maximum phase in terms of α and the frequency at the maximum phase. (5%)

(c) In Fig. 5, if the static velocity error coefficient K_v and the Bode diagram of open-loop transfer function for the system with $G_c(s) = 1$ are given, discuss the effects on K_v and the Bode diagram of open-loop transfer function when the compensator is applied; i.e., $G_c(s) = D(s)$. (5%)

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6. A control system is given as Fig. 6.
   If define $x_1 = y$ and $x_2 = \dot{y}$, the system can be represented in state-space form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{B}r$$

$$y = \mathbf{C}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

   (a) Find **A**, **B**, and **C**. (5%)
   (b) Find the eigenvalues of **A**. (5%)
   (c) Find the range of k from the eigenvalues obtained in (b) to guarantee the stability of the system. (5%)
   (d) Verify your result in (c) by Routh's stability criterion. (5%)

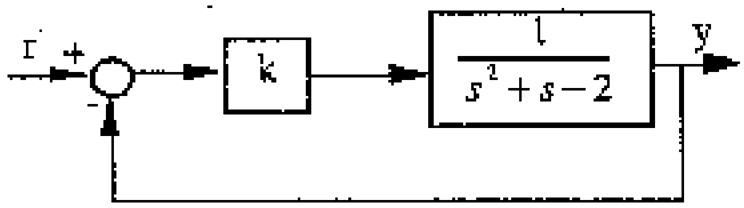

Fig. 6

7. A control system has the structure shown in Fig. 7 where u and y are input and output, respectively, and x_1, x_2, and x_3 are states of the system.
   (a) Test the controllability of the system. (5%)
   (b) Test the observability of the system. (5%)
   (c) If the system is controllable, design the state-feedback control law

$$u = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + v$$

   so that the closed-loop system has poles at -1, -2, and -3. (5%)

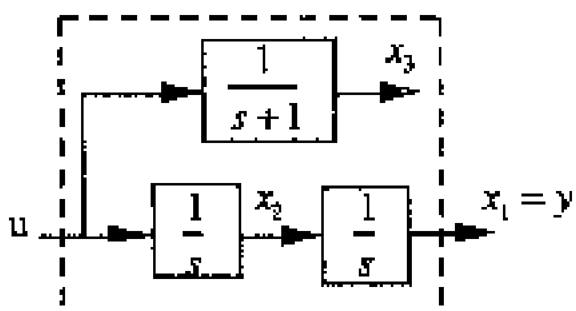

Fig. 7