國立清華大學 106 學年度碩士班考試入學試題

系所班組別：動機系 乙組

考試科目（代碼）：控制系統(1302)

　　*請在【答案卷】作答

Q1 As shown in Figure 1(a &b), the accelerometer is used to measure the car acceleration a. Here $a(s) = G_r R(s) + G_w W(s) + G_v \mathrm{V}(s)$

(1) **(15 pts)** Please find the transfer functions G_r, G_w and G_v as the function of *m, c, k,* and *M* (neglecting gravity).

(2) **(5 pts)** Some engineer ends up a feedback design as shown in Figure 1(b). Based on all given information in problem Q1, **analyze the stability** of this closed-loop system. (You need to explain why you have such a conclusion. **You are only allowed to** use one (or two) rough-drawing root loci and less than 10 words (maybe some more transfer functions) for your explanation.)

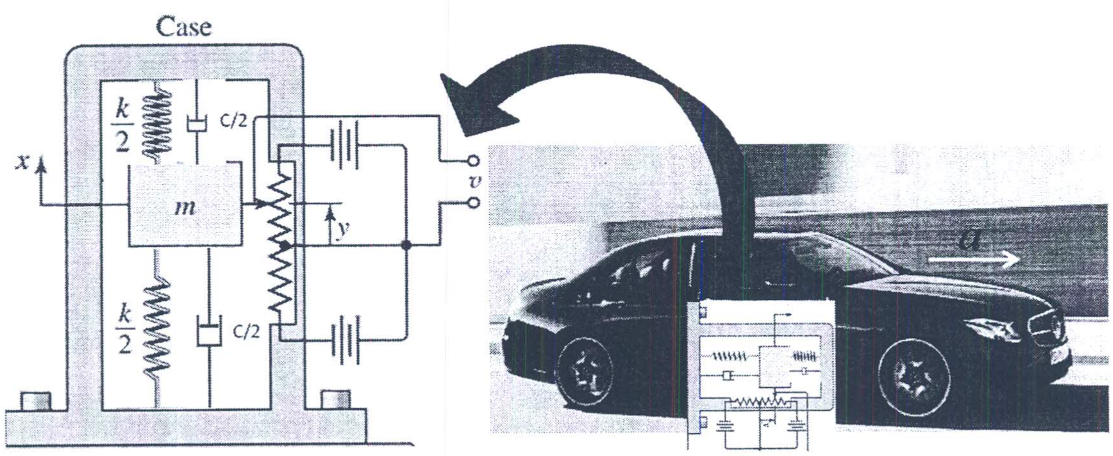

Figure 1 (a)

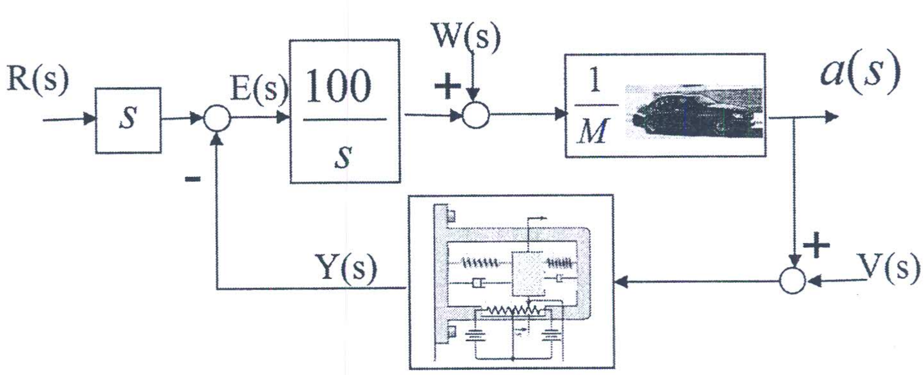

Figure 1 (b)

國立清華大學106學年度碩士班考試入學試題

系所班組別：動機系 乙組

考試科目（代碼）：控制系統(1302)

 *請在【答案卷】作答

Q2 (**<u>5 pts No partial credit!</u>**) Please relate the curve (a), (b), (c) and (d) to the unit step responses of the following systems.

(1) $\dfrac{Y(s)}{R(s)} = \dfrac{2}{s^2 + 0.5s + 2}$

(2) $\dfrac{Y(s)}{R(s)} = \dfrac{-2s + 2}{s^2 + 0.5s + 2}$

(3) $\dfrac{Y(s)}{R(s)} = \dfrac{2}{(s^2 + 0.5s + 2)(s+1)}$

(4) $\dfrac{Y(s)}{R(s)} = \dfrac{2s + 2}{s^2 + 0.5s + 2}$

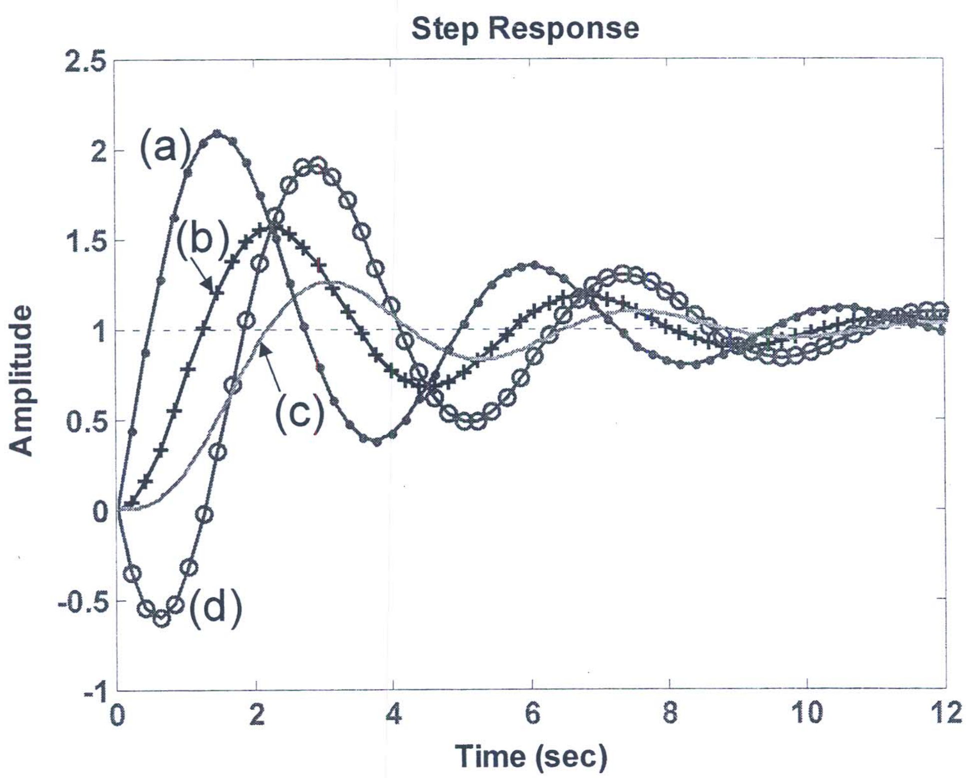

Figure 2

國立清華大學 106 學年度碩士班考試入學試題

系所班組別：動機系 乙組

考試科目（代碼）：控制系統(1302)

　　*請在【答案卷】作答

Q3 Consider the closed-loop control system as shown in Figure 3. Design a PD controller of G_c for G_p to satisfy the closed-loop system with the specification (i) settling time = 2.3 seconds; (ii) overshoot=4.32%.

Please give

(a) your PD controller (K_p , K_d) **(10 pts)**

(b) For a unit step input on W(s) and PD controller from (a), what is the steady-state error for signal on E(s)? **(5 pts)**

(c) Use the ratio of $\frac{K_p}{K_d}$ from (a) results as a fixed value. Make K_d be a tunable value varying from 0 to infinity. Find the range of K_d to make the closed-loop system stable. **(5 pts)**

(d) Continue (c), Draw the root locus. **(5 pts)**

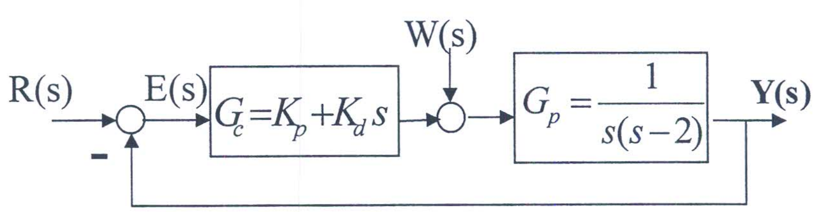

Figure 3

Q4 Consider a unity feedback control as shown in Figure 4. The gain plot of the bode diagram of the loop transfer function $L(s) = C(s)P(s)$ is depicted in Figure 5.

(a) Give an estimation for the gain crossover frequency ω_c ? **(5 pts)**

(b) Assume $L(s)$ is a stable system with minimum-phase zero(s), sketch the Nyquist diagram. What is the phase margin ϑ_m? **(5 pts)**

國立清華大學106學年度碩士班考試入學試題

系所班組別：動機系 乙組

考試科目（代碼）：控制系統(1302)

 *請在【答案卷】作答

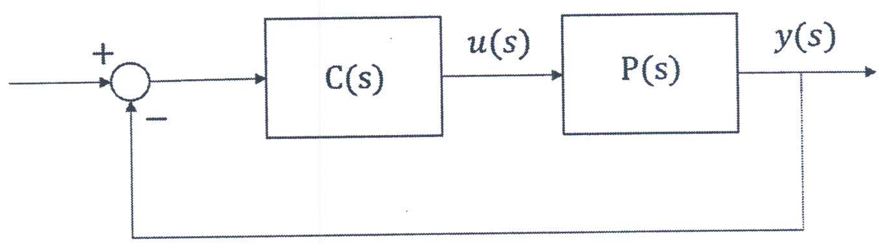

Figure 4

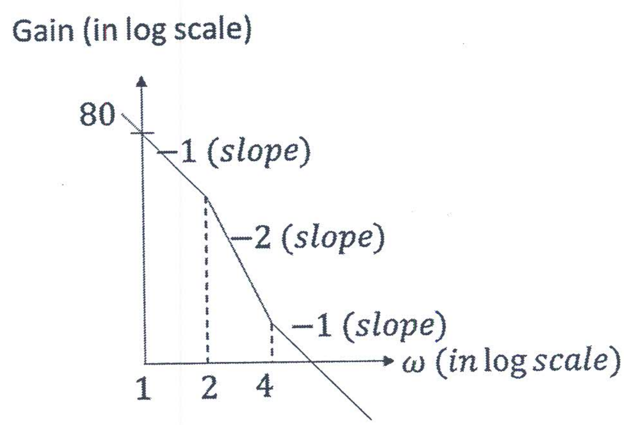

Figure 5

Q5 Consider the feedback control system in Fig. 4 where $P(s)=\dfrac{1}{s(s+1)}$ is the plant.

The design specifications are:

(i) Steady-state error due to ramp input is less than 10%.

(ii) The phase margin is at least 80°.

(iii) For noise sensitivity consideration, $|L(j\omega)|<1$, for $\omega \geq 10 rad/s$ where $L(s)=C(s)P(s)$ is the loop transfer function.

You are given the task to design a controller which satisfies all three specs and yet gives the shortest rise time or highest bandwidth.

(1) At the first try, a proportional control $C(s)=K$ is considered. Show that $K=10$ can satisfy the steady-state error specification. **(2 pts)**

(2) With $C(s)=K=10$, analyze the closed-loop system stability using Nyquist

Theorem. Compute the gain margin and phase margin. **(6 pts)**

(3) Apparently, as you found out in (2), the proportional control does not give a satisfactory phase margin. Try to design a PD controller $C(s) = K_p + K_d s$ which satisfies all the three specifications and yet gives the shortest rise time or highest bandwidth. $K_p, K_d = ?$ What are the phase margin and the gain crossover frequency of the closed-loop system? **(7 pts)**

Q6 In this problem, the design of a dynamic compensator for a satellite plant $\frac{y(s)}{u(s)} = \frac{1}{s^2}$ (as $P(s)$ in Figure 4) is considered.

(a) Convert the plant transfer function into the follow state-space form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= [1 \quad 0]\mathbf{x} \end{aligned}$$

What are the **A** and **B** matrices? **(2 pts)**

(b) (Full-state feedback Design) Assume the state vector **x** is measurable. Design a full-state feedback control law for u to place the control poles at $-\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}} j$. **(4 pts)**

(c) Assume the control signal u is available. Design an observer for the satellite system to estimate the state vector **x** from the output y. The observer poles should be located at $-\frac{5}{2} \pm \frac{5\sqrt{3}}{2} j$. **(5 pts)**

(d) Combine the full-state feedback in (b) and the observer in (c) to form a dynamic compensator. Derive the transfer function (as $C(s)$ in Figure 4) for such a dynamic compensator. **(6 pts)**

(e) Sketch the bode diagrams for $P(s)$ and $C(s)P(s)$. Compute the gain crossover frequency and phase margin. Does $C(s)$ look like a lead compensator or a lag compensator? **(8 pts)**