國立清華大學 104 學年度碩士班考試入學試題

系所班組別：動力機械工程學系碩士班 乙組(電控組)

考試科目（代碼）：控制系統 (1202)

 *請在【答案卷、卡】作答

Q1 A standard Bode plot for a stable, minimum phase plant G is given below.

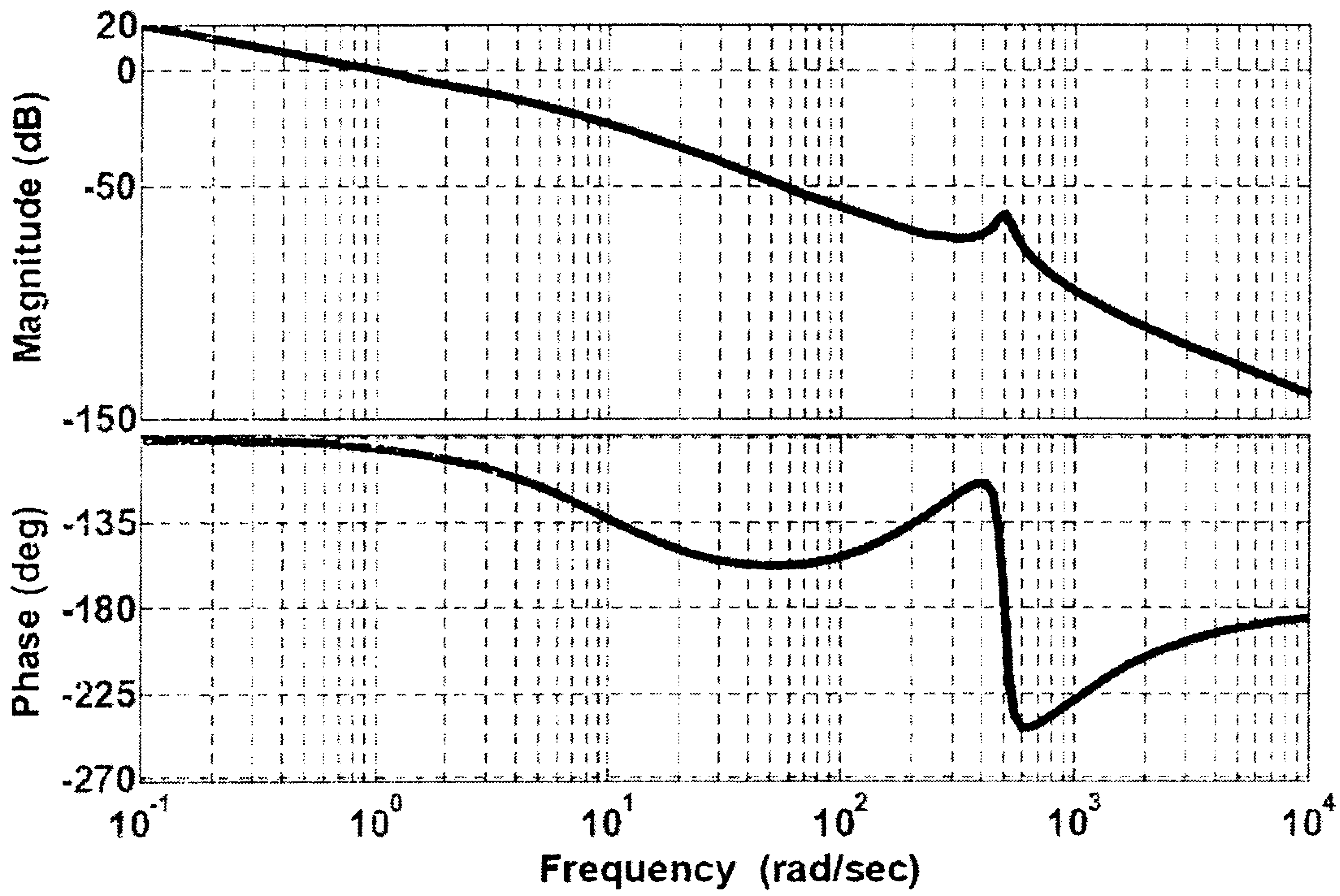

Figure 1

(a) What is the value of gain K such that KG has a phase margin of 45 degrees? (5%)

(b) For the closed-loop system below and $K=1$, what is the steady-state error of $(r(t)-y(t))$ responding a unit ramp input on R? (10%)

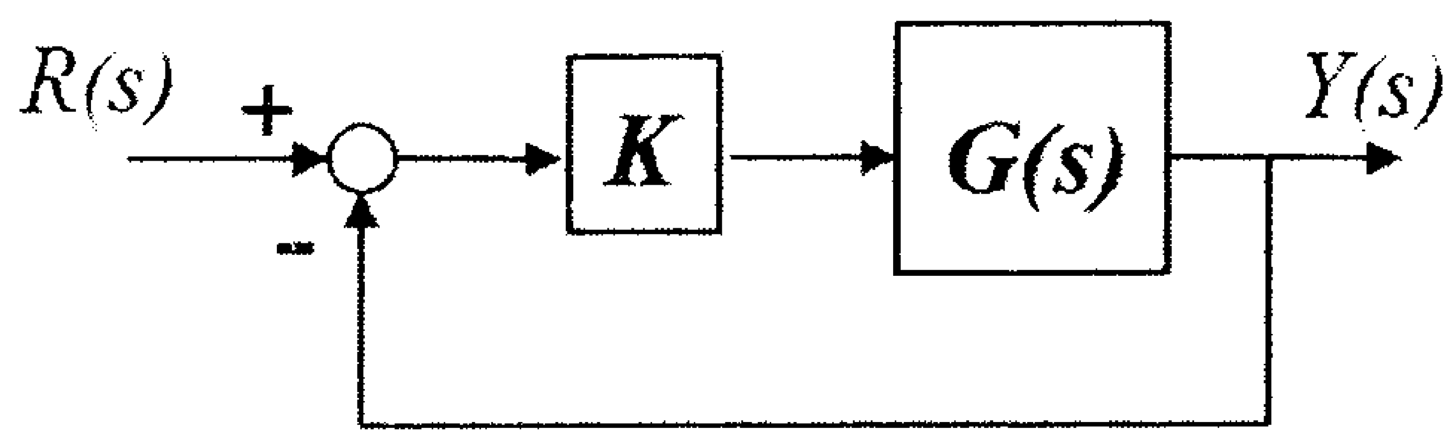

Figure 2

Q2 What is the Gain Margin for the system as shown below? (10%)

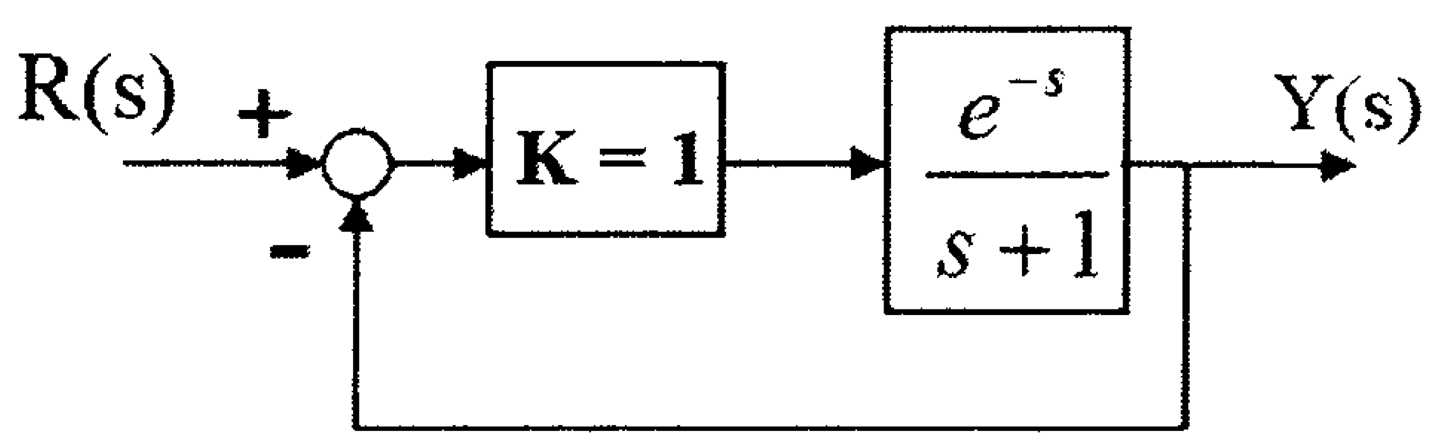

Figure 3

Q3 For the case of K=15 and the block diagram shown below

(a) Draw the Nyquis plot of $Gc(s)Gp(s)$. (10%)

(b) Use Nyquist criterion to decide the closed-loop system stability. (5%)

(c) Considering K *unknown,* **use (b) (Nyquist criterion)** to find the range of K value (K>0) for which the system is stable. (5%)

(d) With your proportional controller (K=15), answer (True or False) for the following statements ((d-1)~(d-5)) by comparing another controller (K=10) -- **(5% no partial credit!)**

(d-1)bandwidth increases; (d-2)overshoot increases; (d-3) settling time increases; (d-4) closed-loop damping increases; (d-5) relative stability increases

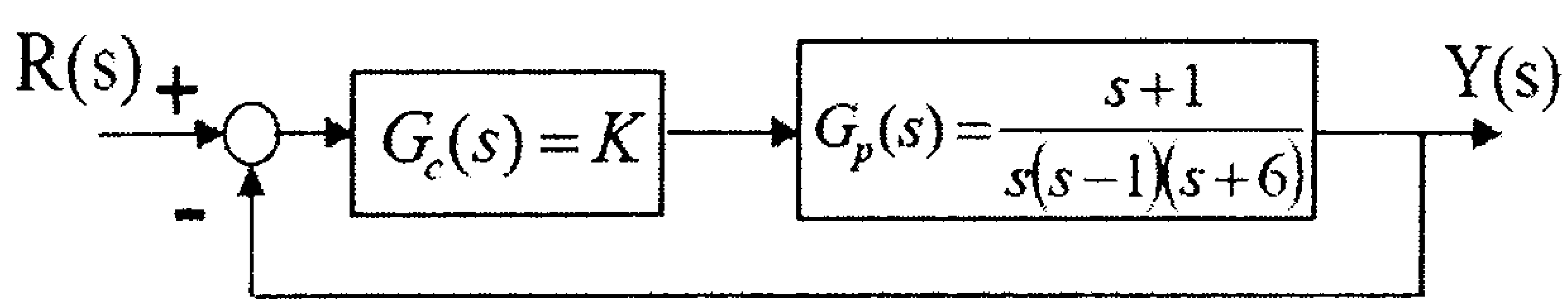

Figure 4

Q4 A second-order plant with adjustable damping ratio, ζ, is described by the transfer function:

$$G_p(s)=\frac{100}{s^2+20\zeta+100} \qquad (1)$$

The plant is to be controlled by a conventional *proportional-plus-integral-plus-derivative* (PID) scheme, with associated parameters k_P, T_i and T_d.

(a) Determine the fixed relationships for the PID parameters T_i and T_d that ensure the controller introduces two zeros at s = - 5rad/s. (5%)

(b) Hence, if the proportional gain is fixed at a value k_P = 1, show that the corresponding closed-loop characteristic equation (CLCE) can be written as:

$$1+(20\zeta)\left[\frac{s^2}{s^3+10s^2+200s+250}\right]=0 \qquad (2)$$

You must show that the denominator term can be factorised as

$(s + 1.326)(s^2 + 8.674s+188.5)$. (6%)

(c) Show that there are two relevant break points at (approximately) s = - 2.587rad/s and s = - 12.67rad/s. (4%)

(d) Sketch the roots' loci of the CLCE from part (b) when the parameter along the curve is a simple function of ζ. (10%)

Q5 A disk-drive read/write head is shown in Figure 5. The system in reality has some flexibility between the read head and the drive motor. The equation of motion of the read head to a torque applied to the base is written as

$$\begin{aligned} I_1\ddot{\theta}_1+k(\theta_1-\theta_2)+b(\dot{\theta}_1-\dot{\theta}_2)&=M_c+M_D \\ I_2\ddot{\theta}_2+k(\theta_2-\theta_1)+b(\dot{\theta}_2-\dot{\theta}_1)&=0 \end{aligned} \qquad (3)$$

Here, M_D is the disturbance torque, the damping is b, M_c is the applied torque, and θ_2 is the output.

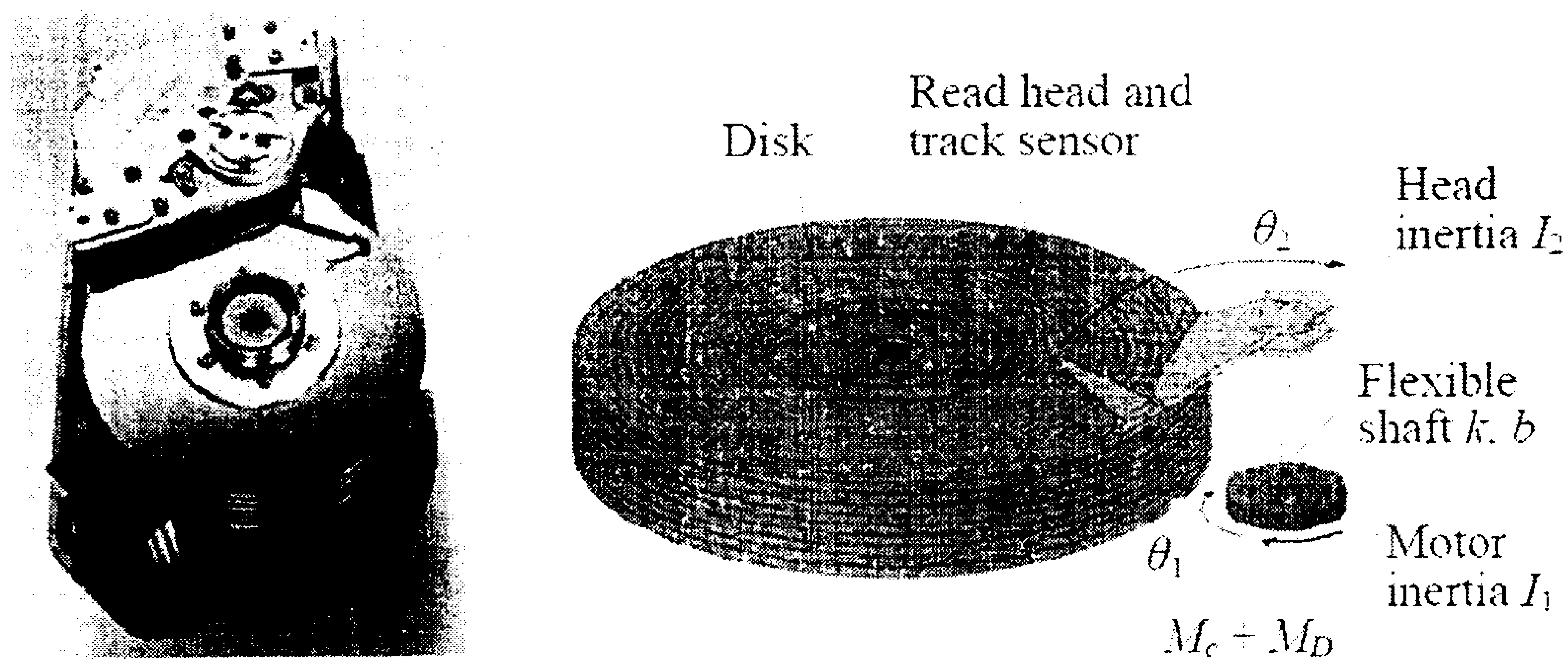

Figure 5

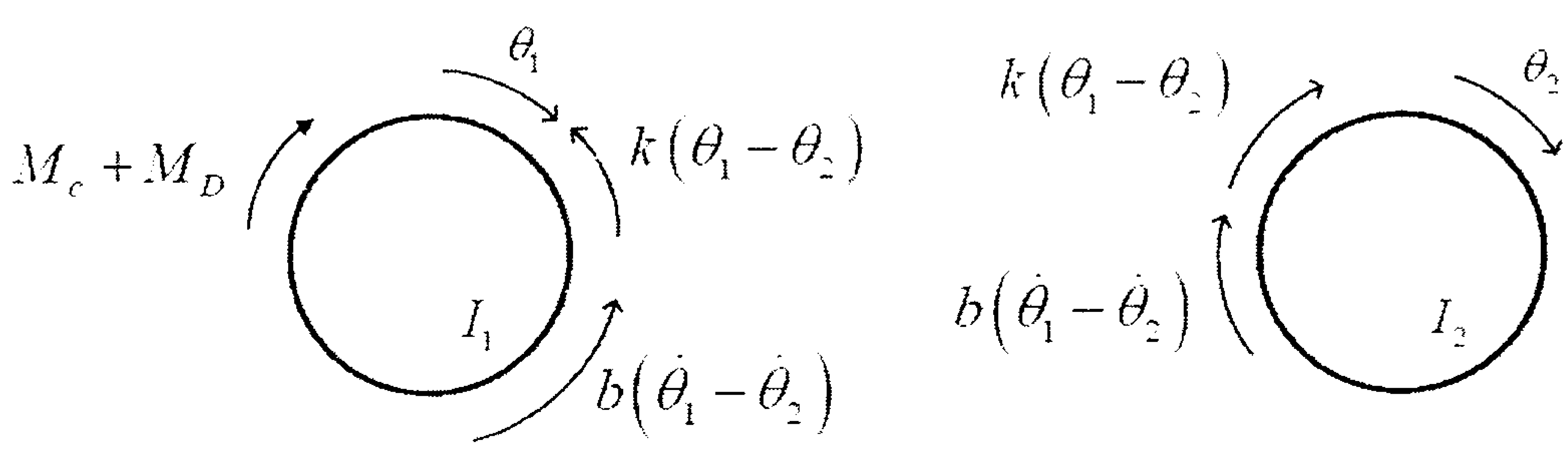

Figure 6

(a) Derive the state-space equation of the system using when the state vector is $\mathbf{x} = [\theta_1\ \dot{\theta}_1\ \theta_2\ \ \dot{\theta}_2]^T$. Indicate the plant matrix **A**, input matrix **B**, output matrix **C**, and the input signal u. (6%) (Hint: The plant matrix **A** is given in Equation (4).)

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{I_1} & -\frac{b}{I_1} & \frac{k}{I_1} & \frac{b}{I_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{I_2} & \frac{b}{I_2} & -\frac{k}{I_2} & -\frac{b}{I_2} \end{bmatrix} \tag{4}$$

(b) When $k = b = I_1 = I_2 = 1$, find the poles of the system and show that two of the poles are at 0, 0. Comment on the stability of the system. (6%) (Hint: stable/unstable/neutrally stable and why?)

(c) Check the controllability of the system. (6%)

(d) If we wish to move the poles of the system to -5, -5, -6, and -6 via a state feedback controller **K**, describe the procedure to design the controller. Do not make any mathematical calculation, just use symbolic equations and notations to introduce and explain the procedure. (7%) (Hint: the open-loop characteristic equation is $\Delta(s) = |s\mathbf{I} - \mathrm{A}|$ and the closed-loop characteristic equation is $\Delta_c(s) = |s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$.)