國立清華大學 103 學年度碩士班考試入學試題

系所班組別：動力機械工程學系碩士班 乙組(電控組)

考試科目（代碼）：控制系統 (1202)

Q1

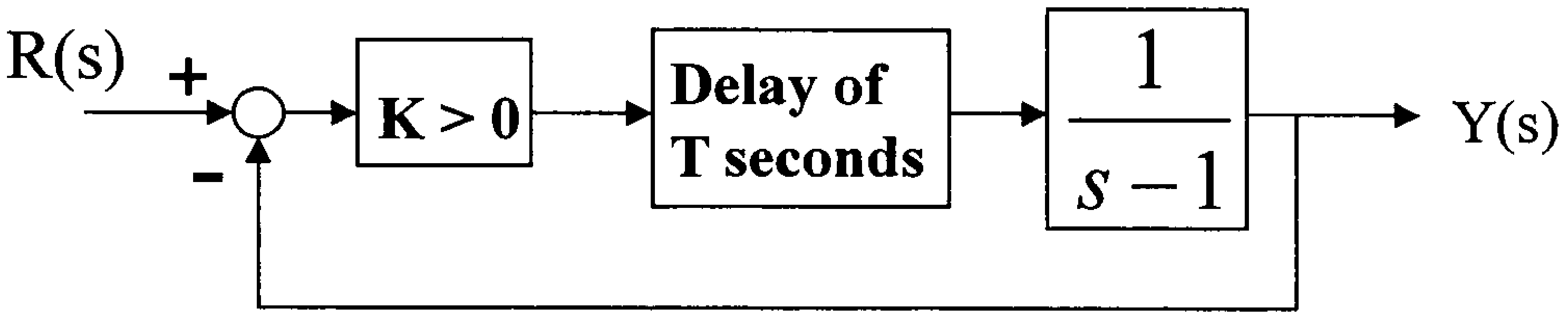

Figure 1

Hint: Use Padé approximant for delay of T seconds: $e^{-Ts} \cong \dfrac{1-\dfrac{Ts}{2}}{1+\dfrac{Ts}{2}}$

(a) If the delay T is less than 1 second, can the system be stabilized and why? If so, what minimum value of K is required to stabilize the system? (5%)

(b) If the delay T is between 1 and 2 seconds, can the system be stabilized and why? If so, what is the range for the required K to stabilize the system? (10%)

Q2

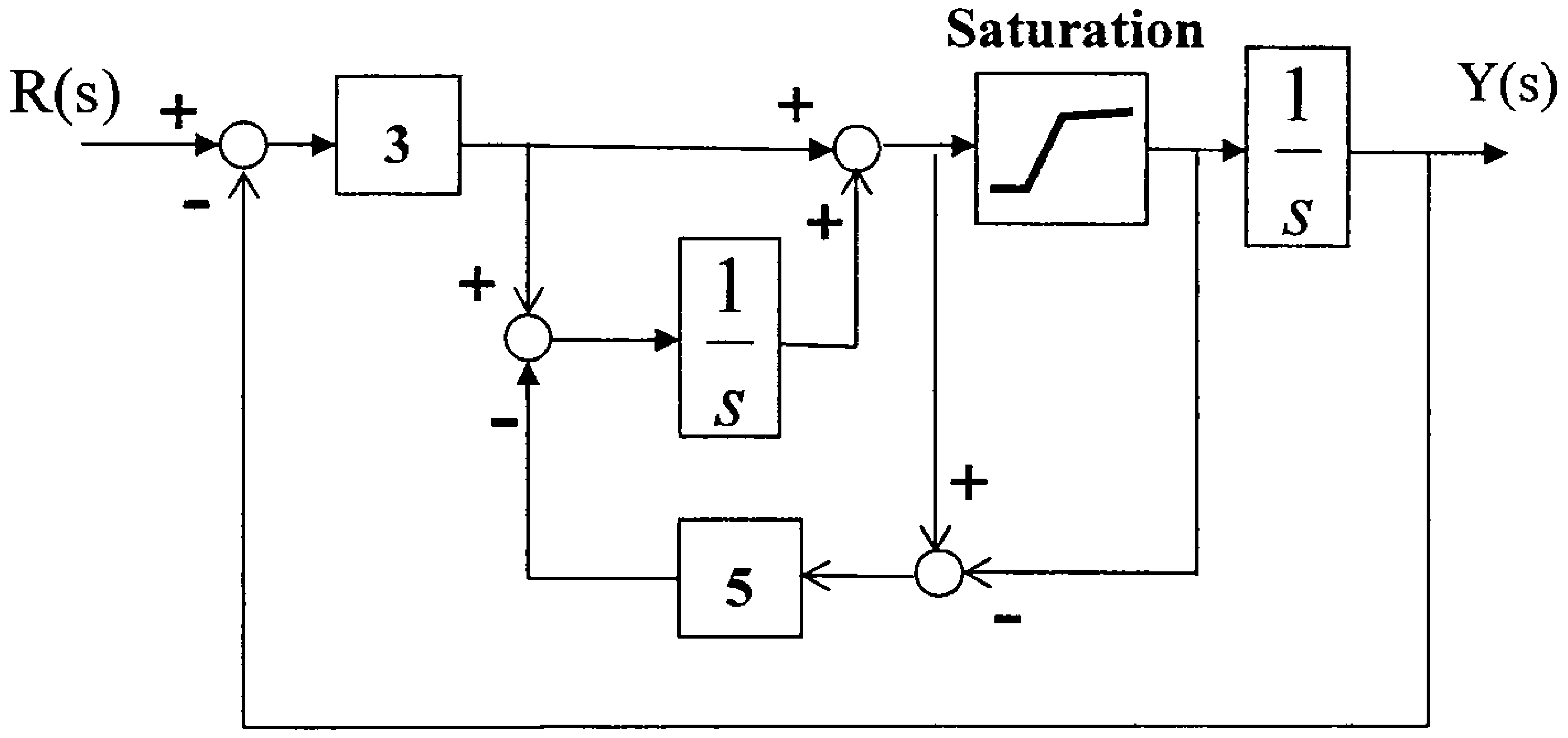

Figure 2 Block diagram illustrating integral anti-windup

If we consider the saturation element shown in Fig. 2 as an equivalent gain K, the root locus for Fig. 2 can be drawn.

(a) Replace the saturation block shown in Fig. 2 with an equivalent gain K, and sketch the root locus with respect to K (K>0). (10%)

(b) Compute the roots of the closed loop characteristic equation if the saturation block has a gain of 1. (5%)

Q3

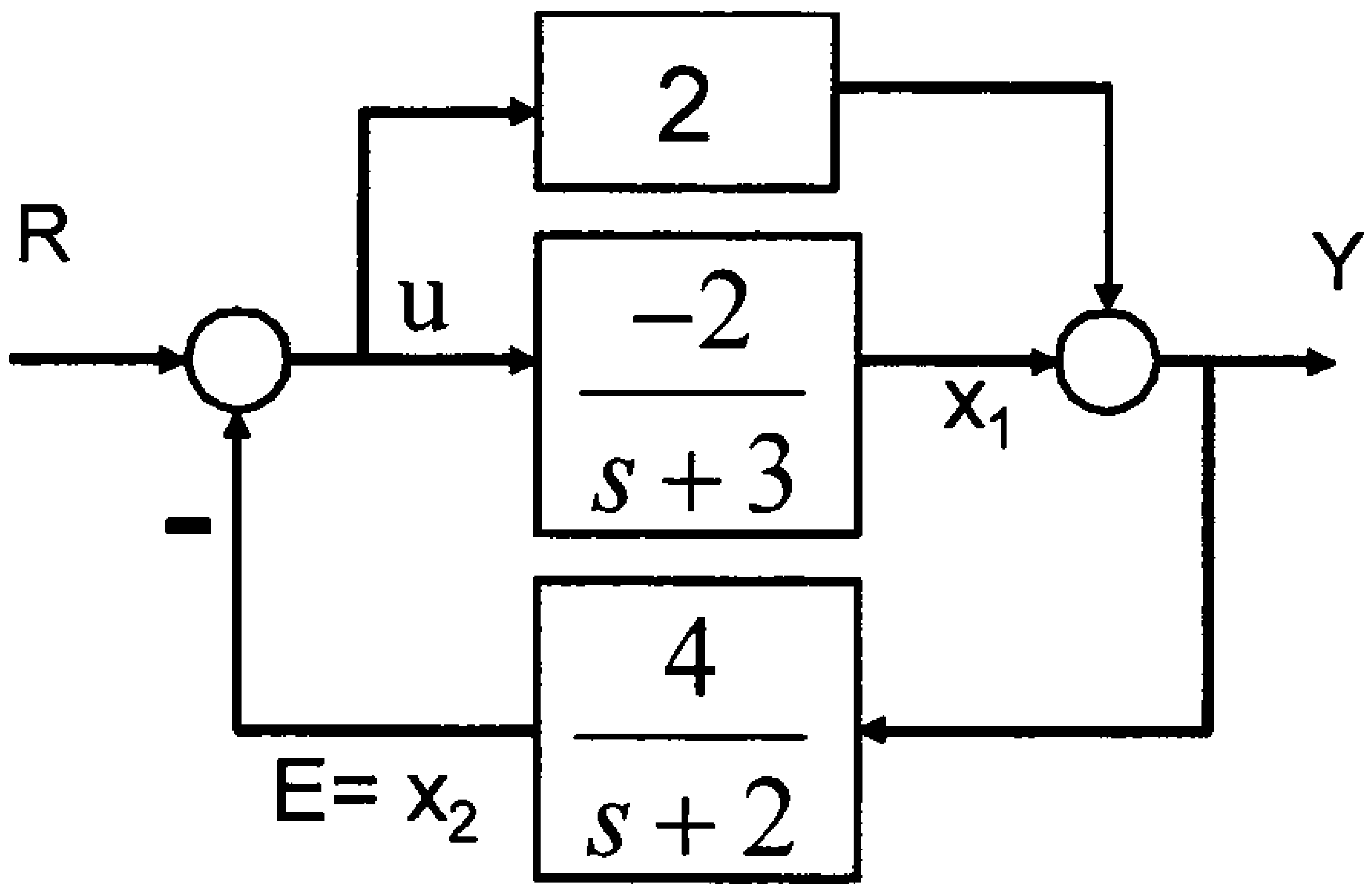

Figure 3

(a) Represent the above system (Fig. 3) in the state-space representation and find the *F*, *G*, *H*, and *J*. (10%)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = F \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + G \bullet r$$

$$y = H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + J \bullet r$$

(b) Check the controllability of the individual closed-loop poles. (controllable/or uncontrollable mode for each closed-loop pole) (5%)

(c) Check the observability of the individual closed-loop poles. (observable/ or unobservable mode for each closed-loop pole) (5%)

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Q4 An electric motor may be modelled using the circuit diagram shown in Fig. 4, where e_b is the back emf voltage and v_i is the input voltage. The back emf is proportional to the speed of the motor $e_b = k_e\dot{\theta}$, where k_e is the emf constant and θ is the rotation of the armature. The torque generated by the motor, T_m, is proportional to the armature current $T_m = k_t i_a$, where k_t is the motor torque constant. The motor armature has second moment of inertia J.

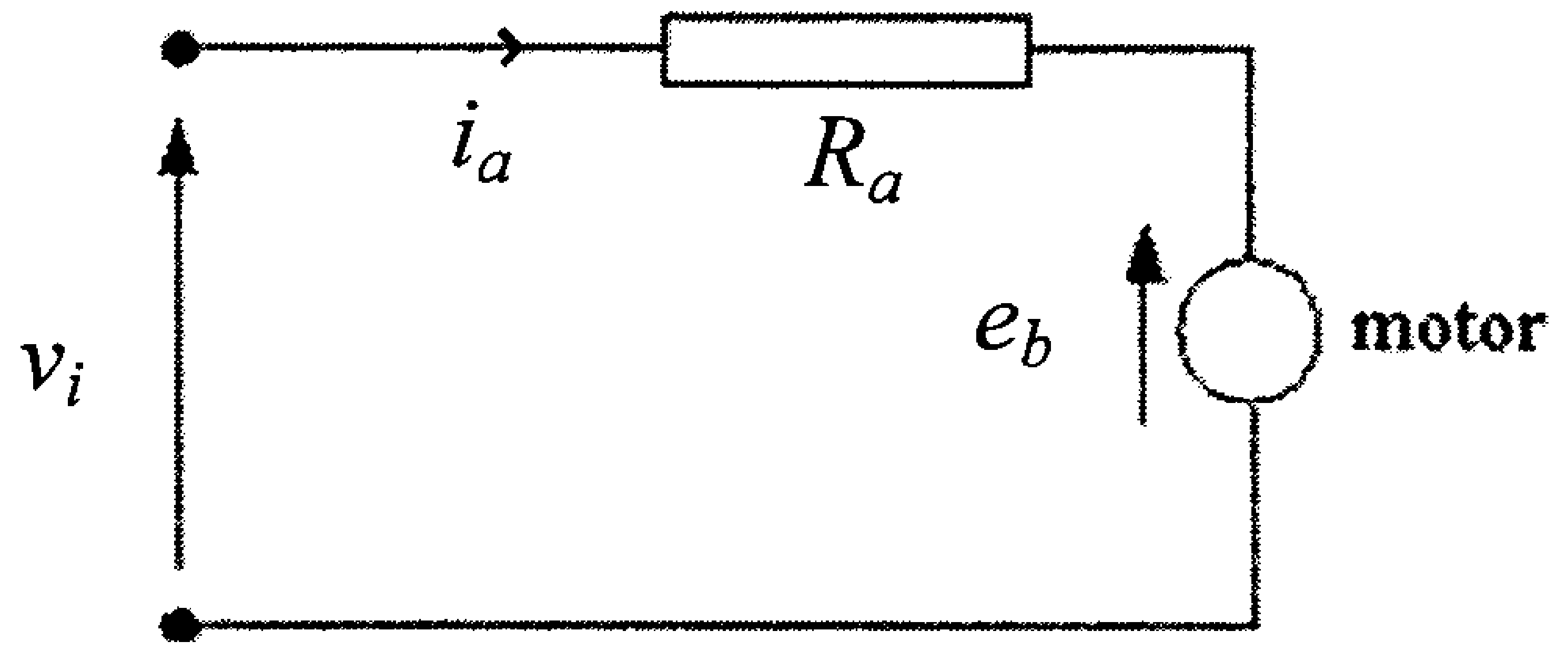

Figure 4

The motor is connected to a rotational spring with rotational spring constant k. The other end of the spring is held rigid. Show that the transfer function between the input voltage v_i and the armature rotation θ is given by:

$$\theta(s) = \frac{\lambda\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} V_i(s)$$

where

$$\lambda\omega_n^2 = \frac{k_t}{R_a J}, \quad \omega_n^2 = \frac{k}{J} \quad \text{and} \quad 2\zeta\omega_n = \frac{k_e k_t}{R_a J}$$

(10 %)

Q5 A plant has transfer function given by:

$$G_P(s) = \frac{50}{s\left(s^2 + 12s + 100\right)}$$

The Bode plots related to this question are shown in Fig. 5.

(a) Write down expressions for the gain and phase of the plant as functions of frequency. (6%)

(b) The plant is to be controlled using a proportional plus derivative feedback controller. Explain why this is a reasonable choice of controller. (2%)

(c) The required open-loop transfer function (OLTF) phase margin is 50° with an OLTF gain crossover frequency of 9rad/s. Find the corresponding controller gains k_p and k_d. (6%)

(d) Write down expressions for the gain and phase of the OLTF. (6%)

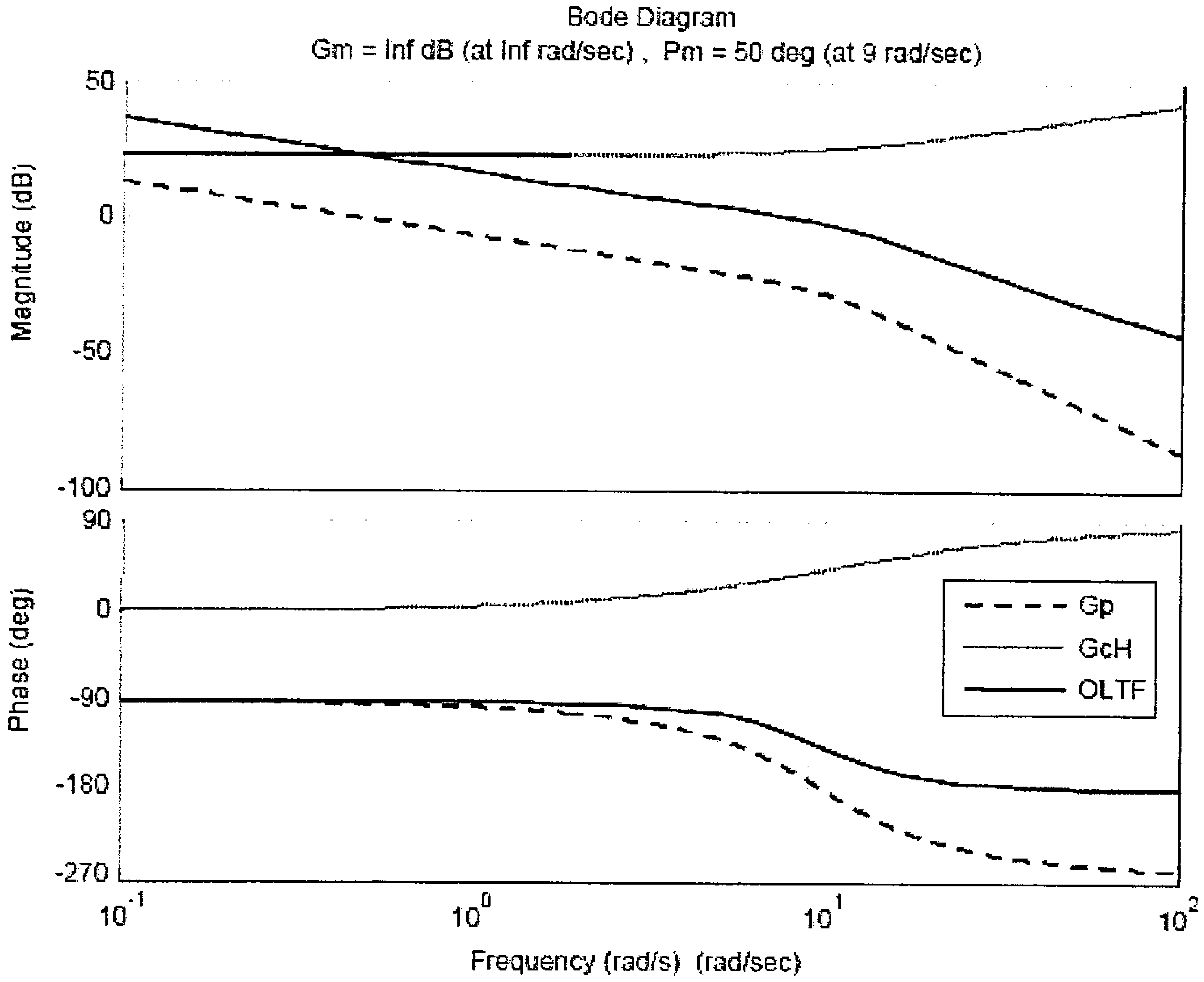

Figure 5

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Q6 Consider the controller transfer function

$$G_c(s) = \frac{K(1+Ts)}{(1+\alpha Ts)}$$

(a) Write down expressions for the gain and phase of the controller as functions of frequency. (4%)

(b) Tables 1 and 2 summarize the bounds/asymptotes and critical points of the gain and phase in the frequency domain. Complete tables for the cases of $\alpha > 1$ (lag) and $0 < \alpha < 1$ (lead). (6%)

Table 1 The case $\alpha > 1$ (lag)

ω	$\lvert G_c(j\omega)\rvert$	$\angle G_c(j\omega)$
0		
$0 < \omega < \infty$		
∞		

Table 2 The case $0 < \alpha < 1$ (lead)

ω	$\lvert G_c(j\omega)\rvert$	$\angle G_c(j\omega)$
0		
$0 < \omega < \infty$		
∞		

(c) Sketch the polar plot when $0 < \alpha < 1$ (lead). Note that the necessary points in Table 2 must be indicated. (5%)

(d) Sketch the frequency plot (including gain and phase plots) when $\alpha > 1$ (lag). Note that the necessary horizontal bounds/asymptotes in Table 1 must be indicated. (5%)