

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：動力機械工程學系甲組(熱流組)，乙組(電控組)，丙組(固體與奈微米力學組)，丁組(設計、製造組)

考試科目 (代碼)：工程數學(1003,1103,1203,1303)

共 2 頁，第 1 頁

\*請在【答案卷、卡】作答

1. Find the inverse Laplace transform of the following function (10%)

$$F(s) = \frac{e^{-2s}}{s^2 + 2s + 3} + 2e^{-3s}$$

2. Find the Laplace transform of the following function (10%)

$$g(t) = \begin{cases} 0 & 0 \leq t < 1 \\ t-1 & 1 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$$

3. Find the most general solution to the ODE (10%)

$$3 \frac{dy}{dx} - y = xe^{x/3}$$

What is the unique solution satisfying  $y(0) = 1$ ?

4. Find the least-square solution of  $\mathbf{Ax} = \mathbf{b}$ , with  $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -1 & 2 \end{bmatrix}$ ,  $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  (10%)

5. Find the expression for the plane tangential to  $xy^3z^2 = 4$  at the point  $(1, 1, 2)$ . (10%)

6. Solve the following problem of partial differential equation (10%)

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + f(x, t)$$

$$T(0, t) = g(t), \quad T(L, t) = h(t)$$

$$T(x, 0) = p(x)$$

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：動力機械工程學系甲組(熱流組)，乙組(電控組)，丙組(固體與奈微米力學組)，丁組(設計、製造組)

考試科目 (代碼)：工程數學(1003,1103,1203,1303)

共 2 頁，第 2 頁

\*請在【答案卷、卡】作答

7. Expand  $f(x) = x^2$  for  $0 < x < L$  (15%)

- (a) in a sine series
- (b) in a cosine series
- (c) in a Fourier series

8. Evaluate the following integrals, where  $C$  is the circle  $x^2 + y^2 = 4$  (counterclockwise)

(a)  $\oint_C \bar{z} dz$  (8%)

(b)  $\oint_C z^2 (e^{-z} + e^{1/z}) dz$  (8%)

9. Show that  $\cosh x \sin y$  is a harmonic function and find its conjugate harmonic function. (9%)