96 學年度材料科學工程學系碩士班入學考試

科目 理工測驗二　　科目代碼 0602　　共 16 頁

注意事項：

1.本科共有 80 題，皆為 5 選 1 的單選題；答對一題得 2.5 分，答錯一題倒扣 0.625 分，未答者不計分。

2.作答時，請以 2B 鉛筆在【答案卡】上畫卡。

3.考試時間為 100 分鐘。

1. $\mathbf{A} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}; \mathbf{B} = \hat{\mathbf{j}} + \hat{\mathbf{k}}; \mathbf{C} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$. Find the volume of the parallelepiped defined by **A**, **B**, and **C**.

   (a) 3 (b) 8 (c) 2 (d) 4 (e) none of the above.

2. For $x = x(u,v),\ y = y(u,v)$, which of the following equation is (in general) correct? (a) $\dfrac{\partial u}{\partial x} = \left(\dfrac{\partial y}{\partial v}\right)^{-1}$

   (b) $\dfrac{\partial(u,v)}{\partial(x,y)} = \left(\dfrac{\partial(x,y)}{\partial(u,v)}\right)^{-1}$ (c) $\dfrac{\partial u}{\partial x} = \left(\dfrac{\partial x}{\partial u}\right)^{-1}$ (d) $\dfrac{\partial(u,x)}{\partial(v,y)} = \left(\dfrac{\partial(v,y)}{\partial(u,x)}\right)^{-1}$ (e) none of the above.

3. If $\mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z$ and $r^2 = x^2 + y^2 + z^2$, then $\nabla \cdot \mathbf{r} f(r) =$ (a) $f(r) + r\dfrac{df}{dr}$ (b) $f(r) + \dfrac{1}{r}\dfrac{df}{dr}$

   (c) $3f(r) + r\dfrac{df}{dr}$ (d) $3f(r) + \dfrac{1}{r}\dfrac{df}{dr}$ (e) none of the above.

4. Consider a space curve *C*, parametrized by $x = x(s)$, $y = y(s)$, and $z = z(s)$, where s is the arc length along *C*. Evaluate the directional derivative du/ds, at the designated point *P*, in the direction of the vector **v**.

   For $u = x^2 + y + z^3$, $P = (a, b, c)$ and $\mathbf{v} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{k}}$, the value is (a) $2x\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3z^2\hat{\mathbf{k}}$ (b) $6x\hat{\mathbf{i}} + 6z^2\hat{\mathbf{k}}$

   (c) $2a + 3c^2 + 1$ (d) $6a + 6c^2$ (e) none of the above.

5. If an even function $f(x)$ is expanded in Fourier series, $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$, which of the following statement is incorrect? (a) $b_n = 0$ (b) $m \neq n \to \int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = 0$

   (c) $a_n = \dfrac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = 0$ (d) $n \neq 0 \to \int_{-\pi}^{\pi} \cos(nx)\cos(nx)dx = \pi$

   (e) $n = 0 \to \int_{-\pi}^{\pi} \cos(nx)\cos(nx)dx = 2\pi$.

6. Denote the Fourier convolution of functions $f(x)$ and $g(x)$ as $f * g \equiv \int_{-\infty}^{\infty} f(x-\xi)g(\xi)d\xi$. Let $F\{f\} = \hat{f}(\omega)$ and $F\{g\} = \hat{g}(\omega)$ Which of the following equations is correct? (a) $F\{f * g\} = \hat{f}\hat{g}$

   (b) $F\{f * g\} = \hat{f} * \hat{g}$ (c) $F\{fg\} = \hat{f} * \hat{g}$ (d) $F\{fg\} = F(f)F(g)$ (e) none of the above.

7. Consider a surface S represented parametrically by equation $\mathbf{R}(u,v) = x(u,v)\hat{\mathbf{i}} + y(u,v)\hat{\mathbf{j}} + z(u,v)\hat{\mathbf{k}}$. $\mathbf{R}_u$ and $\mathbf{R}_v$ are continuous, and $\mathbf{R}_u \times \mathbf{R}_v \neq 0$ Which of the following equations is incorrect?

(a) normal vector $\hat{\mathbf{n}} = \dfrac{\mathbf{R}_u \times \mathbf{R}_v}{\|\mathbf{R}_u \times \mathbf{R}_v\|}$ (b) area element $dA = \|\mathbf{R}_u \times \mathbf{R}_v\| du dv$

(c) equation for tangent plane τ at point P ($\mathbf{R}_P$) is $(\mathbf{R} - \mathbf{R}_p) \cdot \hat{\mathbf{n}} = 1$

(d) area of the surface S $A = \iint_R \|\mathbf{R}_u \times \mathbf{R}_v\| dv du$

(e) if $\mathbf{R}_u \cdot \mathbf{R}_v = 0$, the curvilinear coordinates u and v are orthogonal.

8. **u**, **v**, and **w** are vectors. Which of the following statements is correct?

(a) $(\mathbf{u} \cdot \mathbf{v})' = \mathbf{u}' \cdot \mathbf{v}'$ (b) $(\mathbf{u} \times \mathbf{v})' = \mathbf{u} \cdot \mathbf{v}' + \mathbf{u}' \cdot \mathbf{v}$ (c) $(\mathbf{u} \cdot \mathbf{v} \times \mathbf{w})' = \mathbf{u}' \cdot \mathbf{v} \times \mathbf{w} + \mathbf{u} \cdot \mathbf{v}' \times \mathbf{w} + \mathbf{u} \cdot \mathbf{v} \times \mathbf{w}'$

(d) $[\mathbf{u} \times (\mathbf{v} \times \mathbf{w}]' = \mathbf{u}' \times \mathbf{v} \times \mathbf{w} + \mathbf{u} \times \mathbf{v}' \times \mathbf{w} + \mathbf{u} \times \mathbf{v} \times \mathbf{w}$ (e) none of the above.

9. The Fourier integral representation of function $f(x) = e^{-kx}$ when $x > 0$, $k > 0$, and $f(-x) = f(x)$ is

(a) $\dfrac{k}{\pi}\int_0^\infty \dfrac{\sin wx}{k^2 + w^2} dw$ (b) $\dfrac{2k}{\pi}\int_0^\infty \dfrac{\cos wx}{k^2 + w^2} dw$ (c) $\dfrac{2k}{\pi}\int_0^\infty \dfrac{\cos wx}{k + w} dw$

(d) $\dfrac{2k}{\pi}\int_0^\infty \dfrac{\sin wx}{k + w} dw$ (e) $\dfrac{k}{2\pi}\int_0^\infty \dfrac{\sin wx}{k^2 + w^2} dw$.

10. The cylindrical coordinates (r, θ, z) and the Cartesian coordinates (x, y, z) are related by $r^2 = x^2 + y^2$, $\theta = \tan^{-1}(y/x)$, and $z = z$. Which of the following equations is correct?

(a) $\dfrac{d\hat{\mathbf{e}}_r}{d\theta} = -\hat{\mathbf{e}}_\theta$ (b) $\dfrac{d\hat{\mathbf{e}}_\theta}{d\theta} = \hat{\mathbf{e}}_r$ (c) the position vector $\mathbf{R} = r\hat{\mathbf{e}}_r + z\hat{\mathbf{e}}_z$

(d) the velocity vector is $\mathbf{v}(t) = \dot{r}\hat{\mathbf{e}}_r + \dot{z}\hat{\mathbf{e}}_z$ (e) none of the above.

11. Let the area of a thin plate represented by $x^2 + y^2 \leq a$ and $x \geq 0$ (half circle). If the density function for the plate is given by $\rho(x,y) = |y|$, find the mass of the plate.

(a) $\dfrac{a^3}{2}$ (b) $\dfrac{a^2}{2}$ (c) $\dfrac{a^3}{3}$ (d) $\dfrac{\pi a^3}{2}$ (e) $\dfrac{2a^3}{3}$.

12. Granted sufficient differentiability for the following functions (f) and vectors (**u** and **v**). Which of the following equations is wrong? (a) $\nabla \times (\mathbf{u} + \mathbf{v}) = \nabla \times \mathbf{u} + \nabla \times \mathbf{v}$ (b) $\nabla \cdot (\nabla \times \mathbf{v}) = 0$

(c) $\nabla \times (f\mathbf{v}) = \nabla f \times \mathbf{v} + f\nabla \times \mathbf{v}$ (d) $\nabla \times (\nabla f) = 0$ (e) $\nabla \times (\mathbf{u} \times \mathbf{v}) = \mathbf{u} \cdot \nabla \times \mathbf{v} - \mathbf{v} \cdot \nabla \times \mathbf{u}$

13. Expand the function, $\dfrac{1}{1-z^2}$, in Laurent series with center at $z = 1$ in domain $|z-1| > 2$.

(a) $f(z) = -\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{n+1}} (z-1)^{n-1}$ (b) $f(z) = -\sum_{n=0}^{\infty} \frac{(-2)^n}{(z-1)^{n+2}}$ (c) $f(z) = -\sum_{n=0}^{\infty} \frac{n!}{(z-1)^{n+2}}$

(d) $f(z) = -\sum_{n=0}^{\infty} \frac{1}{n!} \frac{(z-1)^{n+2}}{(-2)^{n+1}}$ (e) none of the above.

14. For a vibrating rectangular membrane with dimension a in x direction and b in y direction and the boundary conditions of no deflection at the four corners, the solution for the deflection has the form of $w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} H_{mn} X_{mn}(x,y) T_{mn}(t)$. Then, $X_{mn}(x,y)$ is equal to (a) $\sin\frac{n\pi x}{a}\cos\frac{m\pi y}{b}$

(b) $\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$ (c) $\sin\frac{m\pi x}{2a}\sin\frac{n\pi y}{2b}$ (d) $\sin\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b}$ (e) $\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$.

15. Which of the following equations that conformally map □i, 0, i onto ∞, -1, 0, respectively?

(a) $w = \frac{1}{z}$ (b) $w = \frac{1}{z+1}$ (c) $w = \frac{z-i}{z+i}$ (d) $w = \frac{z+1}{z-i}$ (e) $w = \frac{z+i}{z-1}$.

16. Complex integration of function $f(z)$. $\int_C f(z)dz$ where $f(z) = u + iv; z = a + ib$. Which of the following equations is correct?

(a) the real part of the integration is $\int_C (udx + vdy)$

(b) the real part of the integration is $\int_C (udy + vdx)$

(c) the imaginary part of the integration is $\int_C (vdy + udx)$

(d) the imaginary part of the integration is $\int_C (vdx + udy)$

(e) none of the above

17. Evaluate $S = \sqrt{i} - \sqrt{-i}$ in the principle argument of the complex. S =

(a) 1 (b) $\sqrt{2}$ (c) $2e^{-i\pi/4}$ (d) $2e^{-i3\pi/4}$ (e) 0.

18. Integrate $\dfrac{1}{(4z+i)}$ in the counterclockwise sense around the unit circle of the complex plane. The answer is (a) $i/4$ (b) $\pi i/4$ (c) $\pi/4$ (d) $\pi i/2$ (e) 0.

19. Evaluate the integral $I=\int_0^\infty \frac{\sqrt{x}dx}{1+x^2}$. $I=$ (a) π (b) $2\pi I$ (c) $\pi/(2i)$ (d) $\pi/2$ (e) none of the above.

20. For a complex function $f(z)=u(x,y)+iv(x,y)$ defined throughout some neighborhood of a point $z_0=x_0+iy_0$ to be differentiable at z_0, then, which of the following equations is incorrect?

(a) $\frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ (c) $f'=u_x+iv_x$ (d) $f'=v_y+iv_x$ (e) $f'=u_x-iu_y$.

21. The general solution of $y'-4y=4y^2$ can expressed by the form of $y=\frac{Ae^{Bx}}{C-e^{Bx}}$, where C is arbitrary.

If so, ***A*** is (a) -2 (b) -1 (c) 1 (d) 2 (e) none of the above

22. (Cont) Following the previous question, ***B*** is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

23. The exact solution of $y'=y+2x-x^2$; $y(0)=1$ $(0\le x<\infty)$ is $y(x)=x^A+e^{Bx}$. Thus, the sum of **A** and **B** is (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

24. (Cont) If one uses Euler method to calculate the numerical solution of $y'=y+2x-x^2$ whose step size is 0.1, then $y(x=0.1)$ is

(a) 1.10 (b) 1.11 (c) 1.12 (d) 1.13 (e) 1.14

25. (Cont) Again, if one uses midpoint method to calculate the numerical solution of $y'=y+2x-x^2$ whose step size is 0.1, then $y(x=0.1)$ is rounded to be,

(a) 1.10 (b) 1.11 (c) 1.12 (d) 1.13 (e) 1.14

26.The general solution of $x^2y''+xy'+(\lambda^2x^2-\nu^2)y=0$ is

(a) $c_1J_u(\lambda x)+c_2Y_u(\lambda x)$ (b) $c_1J_\lambda(ux)+c_2Y_\lambda(ux)$ (c) $c_1J_u(x)+c_2Y_u(x)$

(d) $c_1J_\lambda(x)+c_2Y_\lambda(x)$ (e) none of the above

(Ps: Here $J(x),Y(x)$ are Bessel functions, and c_1,c_2 are the constants of integration.)

27.The gamma function is expressed as following $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ ($x>0$). Find $\Gamma(4)$ accordingly,

(a) 2 (b) 4 (c) 6 (d) 24 (e) 120

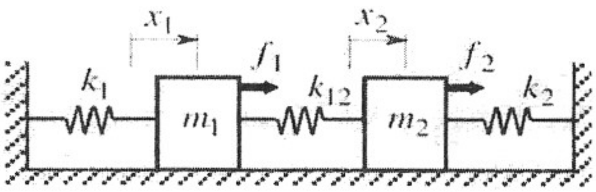
28.The solution of the inverse Laplace transform of $\dfrac{5}{(s+1)(3s+2)}$ can be presented as $A(e^{Bt}-e^{Ct})$,

where A is positive. A is (a)1/3 (b) 2/3 (c) 1 (d) 5/3 (e) 5

29.(Cont) In addition, the sum of B and C in the preceding question is

(a) -5/3 (b) -1/3 (c) 0 (d) 1/3 (e) 5/3

30.Consider the system of two masses suggested to forces $f_1(t)$ and $f_2(t)$ retrained laterally by springs and supported vertically by a frictionless table as shown below. The equations of motion for two masses are $m_1x_1''+(k_1+k_{12})x_1-k_{12}x_2=f_1(t)$ and $m_2x_2''-k_{12}x_1+(k_{12}+k_2)x_2=f_2(t)$, respectively.



Supposing $m_1=m_2=k_1=k_{12}=k_2=1$, the homogeneous solutions of $x_1(t)$ is,

(a) $\alpha\sin(t+\phi_1)+\beta\cos(t+\phi_2)$ (b) $\alpha\sin(t)+\beta\sin(\sqrt{3}t)$ (c) $\alpha\sin(t+\phi_1)$

(d) $\alpha\sin(\sqrt{3}t+\phi_1)$

(e) $\alpha\sin(t+\phi_1)+\beta\sin(\sqrt{3}t+\phi_2)$

(Ps: Here $\alpha,\beta,\phi_1,\phi_2$ are the constants of integration.)

31.(Cont) Moreover, in the two-mass system aforementioned, the homogeneous solution of $x_2(t)$ is,

(a) $\alpha\sin(t+\phi_1)+\beta\cos(t+\phi_2)$ (b) $\alpha\sin(\sqrt{3}t+\phi_1)+\beta\sin(\sqrt{3}t+\phi_2)$ (c) $\alpha\sin(t+\phi_1)$

(d) $\alpha\sin(\sqrt{3}t+\phi_1)$ (e) $\alpha\sin(t+\phi_1)-\beta\sin(\sqrt{3}t+\phi_2)$

(Ps: Here $\alpha,\beta,\phi_1,\phi_2$ are the constants of integratio n.)

32.For a coupled system $x'=x+y; x(0)=0$ and $y'=x+y+e^{3t}; y(0)=0$, solve $x(t)$ and $y(t)$, respectively.

First of all, $x(t)$ is,

(a) $x(t)=\frac{1}{3}e^{3t}-\frac{1}{2}e^{2t}+\frac{1}{6}$ (b) $x(t)=\frac{1}{2}e^{3t}-\frac{1}{3}e^{2t}+\frac{1}{6}$ (c) $x(t)=\frac{1}{3}e^{3t}-\frac{1}{2}e^{2t}-\frac{1}{6}$

(d) $x(t)=\frac{1}{2}e^{3t}-\frac{1}{3}e^{2t}-\frac{1}{6}$ (e) $x(t)=\frac{2}{3}e^{3t}-\frac{1}{2}e^{2t}+\frac{1}{6}$

33.(Cont) $y(t)$ is,

(a) $y(t)=\frac{1}{3}e^{3t}-\frac{1}{2}e^{2t}+\frac{1}{6}$ (b) $y(t)=\frac{1}{3}e^{3t}-\frac{1}{2}e^{2t}+\frac{1}{6}$ (c) $y(t)=\frac{1}{3}e^{3t}-\frac{1}{2}e^{2t}-\frac{1}{6}$

(d) $y(t)=\frac{1}{2}e^{3t}-\frac{1}{3}e^{2t}-\frac{1}{6}$ (e) $x(t)=\frac{2}{3}e^{3t}-\frac{1}{2}e^{2t}-\frac{1}{6}$

34.If A is $\begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 3 & 3 & 6 \\ 1 & 4 & 5 & 9 \end{bmatrix}$, then the rank of A is

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

35.Find the following determinant. You can use **co-factor expansion** or/and **triangularization** to do so.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 0 & 1 & -3 & 5 \end{vmatrix}$$

(a) 0 (b) 206 (c) -206 (d) 5 (e) -5

36.The inverse matrix of $\begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \\ 1 & 5 & -2 \end{bmatrix}$ is

(a) $\frac{1}{57}\begin{bmatrix} -22 & -1 & 9 \\ 4 & -5 & -12 \\ -1 & 13 & -3 \end{bmatrix}$ (b) $\frac{1}{57}\begin{bmatrix} -22 & -1 & 9 \\ 4 & -5 & -12 \\ -1 & -13 & 3 \end{bmatrix}$ (c) $\frac{1}{57}\begin{bmatrix} 22 & -1 & 9 \\ 4 & -5 & 12 \\ -1 & -13 & 3 \end{bmatrix}$

(d) $\frac{1}{57}\begin{bmatrix} 22 & -1 & 9 \\ 4 & 5 & -12 \\ -1 & -13 & 3 \end{bmatrix}$ (e) $\frac{1}{57}\begin{bmatrix} -22 & 1 & 9 \\ 4 & 5 & -12 \\ 1 & 13 & 3 \end{bmatrix}$

37.For the following matrix $\begin{bmatrix} 3 & 2i \\ -2i & 0 \end{bmatrix}$, what is the sum of its eigenvalues?

(a) 2 (b) 3 (c) 4 (d) 5 (e) none of the above

38.Based on the eigenvalue problem you solve above, what is the dot product of the corresponding eigenvectors?(a) $4i$ (b) 0 (c) -3 (d) 3 (e) none of the above

39.The general solution of the following equation $x^3y''' + x^2y'' - 2xy' + 2y = \frac{2}{x}$ can be expressed as $c_1x^p + c_2x^q + c_3x^r + A\frac{\ln x}{x}$, where c_1, c_2, c_3 are the constants of integration.). What is the sum of p, q, and r?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

40.(Cont) What is A?

(a) -1/2 (b) -1/3 (c) 1/3 (d) 1/2 (e) 1

41. Which statement is not true?

(a)The heat capacity is used when the addition of heat to or withdrawal of heat from the system produces a temperature change.

(b)The concept of heat capacity can be used when a phase change is involved.

(c)The heat capacity is an extensive property.

(d)If the process is carried out at constant volume, all of the heat added is used to raise the temperature of the system.

(e)If the process is carried out at constant pressure, in addition to raising the temperature, the heat added is required to provide the work.

42. Which statement is true?

(a)No work is needed for free expansion.

(b)The internal energy of a gas is independent of volume.

(c)The internal energy of a gas is function only of temperature.

(d)The difference of heat capacities at constant pressure and constant volume is always equal to R.

(e)For solids, the internal energy is independent of volume.

(Questions 43 and 44)

Two moles of an ideal gas are contained adiabatically at 30 atm pressure and 298K. The pressure is suddenly released to 10 atm, and the gas undergoes an irreversible adiabatic expansion as a result of which 2000 joules of work are performed.

43. What is the final temperature of the gas after the irreversible expansion?

(a) 192K (b) 350K (c) 218 K (d) 396 K (e) 77 K

44. What is the created entropy due to the irreversible process (unit: joules/degree)?

(a) +3.7 (b)-7.8 (c)+16.3 (d) -27.9 (e)+5.2 (note: ln 0.73=-0.31; ln 2.19=0.78)

(Questions4 5 and 46)

At a pressure of 1 atm the equilibrium melting temperature of lead is 600K, and at this temperature the latent heat of fusion of lead is 4810 joules per mole. 1 mole of supercooled liquid lead spontaneously freezes at 590 K and 1 atm.

45. Which statement is not true?

(a) If the supercooled liquid lead is heated reversibly from 590 to 600K at 1atm, the entropy change of lead is positive.

(b) If the liquid lead is solidified reversibly at 600K, the entropy change of lead is negative.

(c) If the solid lead is reversibly cooled from 600 to 590 K at 1 atm, the entropy change of lead is negative.

(d) If the liquid lead is solidified reversibly at 600K, the entropy change of lead is equal to the latent heat of freezing divided by the freezing temperature.

(e) The entropy change in the lead for the 1 mole of supercooled liquid lead, which spontaneously freezes at 590 K and 1 atm, is positive due to the irreversible process.

46. Consider the heat entering the constant-temperature heat reservoir at 590K and 1 atm for the freezing process. The enthalpy change in the lead for the 1 mole of supercooled liquid lead, which spontaneously freezes at 590 K and 1 atm, is -4799 joules. What is the entropy change for the heat reservoir at 590K. (unit: joules/degree)

(a) 23.55 (b) -23.55 (c) -8.13 (d) 8.13 (e) 0

47. Which statement is incorrect?

(a) The equilibrium state of a system is simply the most probable of all its possible states.

(b)Configuration entropy is related to the number of ways in which the particles themselves can be mixed over the available position in space.

(c) The relationship between the number of microstates available to the system, Ω, and the entropy of the system is given by $S=k\ln \Omega$.

(d) The total entropy of a system comprises two contributions: the thermal entropy and the configuration entropy.

(e) When a system undergoes a change of state during which it performs work and absorb heat, the magnitudes of the quantities work and heat are minimized when the change of state occurs reversibly.

48. Which statement is incorrect?

(a) The enthalpy of an ideal gas is a function only of temperature.

(b) The free energy of an ideal gas is a linear function of the logarithm of its pressure.

(c) The deviation of real gases from ideality arises from the fact that real gases comprise atoms or molecules of finite volume among which occur finite interactions.

(d) The standard state is that in which the fugacity of the gas is unity at the temperature T.

(e)There is a heat effect involved when the ideal gases are mixed.

49. Which equation is not equal to Cp-Cv (difference of heat capacities at constant pressure and volume) ?

(a) $V\left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial H}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P$　(b) $\left[V - \left(\frac{\partial H}{\partial P}\right)_T\right]\left(\frac{\partial P}{\partial T}\right)_V$　(c) $T\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial P}{\partial T}\right)_V$

(d) $\left(\frac{\partial H}{\partial T}\right)_P + \left(\frac{\partial H}{\partial T}\right)_V + V\left(\frac{\partial P}{\partial T}\right)_V$　(e) $\left(\frac{\partial H}{\partial T}\right)_P - \left(\frac{\partial E}{\partial T}\right)_V$

50. A quantity of supercooled liquid tin is adiabatically contained at 495 K. Calculate the fraction of tin which spontaneously freezes. Given:

ΔH_m (Sn)=7070 joules/mole at T_m=505 K

$C_{p,Sn}$ (l)=34.7-9.2*10^{-3} T joules/degree.mole

$C_{p,\,Sn}$ (s)=18.5+26*10^{-3}T joules/degree.mole

(a) 0.0426　(b) 0.426　(c)0.821　(d)0.0821　(e)0.635

51. Which statement is correct?

(a) The glassy state has an entropy at 0 K equal to zero.

(b) The entropy of any substance is equal to zero at 0 K.

(c) A nonequilibrium concentration of defects in a pure crystalline solid at 0 K can give rise to a nonzero entropy at 0K.

(d) When two elements are mixed to form an alloy at room temperature, the entropy of the system is unchanged.

(e) The variation of ΔG (Gibbs free energy change) with temperature has a slope smaller than zero at 0 K.

(Questions 52-53)

The molar volume of Fe is 7.1 cm^3. The isobaric coefficient of thermal expansion is $3*10^{-5}$ $K^{-1.}$ Assume the volume and the isobaric coefficient of thermal expansion are independent of pressure.

52. What is the increase in the molar enthalpy of Fe resulting from an increase in pressure from 1 to 100 atm at 298 K?
(a) 18 (b) 27 (c) 5 (d) 38 (e) 71 joules

53. What is the change in the molar Gibbs free energy of Fe resulting from an increase in pressure from 1 to 100 atm?
(a) 81.5 (b)70.3 (c) 65.9 (d) 36.2 (e) 19.5 joules

(Questions 54-55)

For H_2O $\Delta H_m = \Delta H_{(s=l)}$=6008 joules at 273 K

$S_{H2O\,(l),298}$=70.08 joules/degree, $S_{H2O(s),298}$=44.77 joules/degree

$C_{p,\,H2O\,(l)}$=75.44 joules/degree and $C_{p,H2O\,(s)}$=38 joules/degree

54. What is the enthalpy difference between water and ice at 473K?
(a) 7604 (b) 1250 (c) 5661 (d) 13496 (e) 2578 joules

55. What is the Gibbs free energy change from ice to water at 273 K?
(a) -214 (b) -15 (c) 0 (d) 148 (e) 7 joules

(Questions 56-58)

The vapor pressure of solid zinc varies with temperature as

ln P (atm)= -15775/T-0.755 ln T +19.25

and the vapor pressure of liquid zinc varies with temperature as

ln P (atm)= -15246/T-1.255 ln T +21.79

The boiling temperature of zinc is 1181 K and the triple-point temperature is 708 K.

56. What is the heat of evaporation of zinc at the boiling temperature?
(a) 114.4 (b) 210.5 (c) 319.3 (d) 428.6 (e) 1184.3 **kjoules/mole**

57. What is the heat of fusion of zinc at the triple-point temperature?
(a) 80.52 (b) 7.34 (c) 68.38 (d) 1.26 (e) 26.59 **kjoules/mole**

58. What is the difference between the heat capacities of solid and liquid zinc?

(a) 12.38　(b) 62.45　(c) 1.89　(d) 24.19　(e) 4.15　joules/degree mole

(Questions 59-60)

Carbon has two allotropes, graphite and diamond. At 298 K and 1 atm, graphite is the stable form. Given the following conditions, and assume the isothermal compressibilities of the two phases are negligible. 9.87 cm^3/atm=1 joule.

$H_{298\,(graphite)}-H_{298\,(diamond)}= -1900$ joules/mole

$S_{298\,(graphite)}=5.73$ joules/degree mole

$S_{298\,(diamond)}=5.73$ joules/degree mole

The density of graphite at 298K is 2.22 gram/cm^3

The density of diamond at 298K is 3.515 gram/cm^3

59. What is the ΔG (Gibbs free energy change) for the phase transformation from graphite to diamond at 298 K?

(a)-1305　(b) -1900　(c) 5730　(d) 2883　(e) 3987 joules/mole

60. What is the pressure which must be applied to graphite at 298K to bring its transformation to diamond?

(a) 270　(b) 14300　(c) 0.03　(d) 5600　(e) 1 atm

Lookup table for some ln *x*

ln2	ln3	ln4	ln5	ln6	ln7	ln8	ln9	ln10
0.69	1.10	1.39	1.61	1.79	1.94	2.08	2.20	2.30

ln 0.2 = ln 2 ? ln 10;　ln(1-x) ~ (-x) for x<<1.

Question 61-64

Component A and B tend to form an ideal solution. The standard molar Gibbs free energy for each component is given as

$$G^o{}_A = 50 - 0.008T, \qquad G^o{}_B = 30 - 0.002T\,;$$

wherein the free energy is in unit of kJ and temperature in Kelvin degree. Answer the following questions, using the look-up table above and assuming R=8 J/K-mole.

61. Which of the following value is the most close to the molar enthalpy $H(T=300\text{ K})$ for the solution with $x_A=0.3$?

(a) 0 kJ　(b) 16 kJ　(c)26 kJ　(d) 36 kJ　(e) 46 kJ

62. Which of the following value is the most close to $\Delta G^M(T= 300\text{ K})$ for the solution with $x_A=0.3$?
(a) 0 kJ (b) -1.5 kJ (c) -3 kJ (d) -15 kJ (e) -30 kJ

63. By raising the temperature from 300 K to 600 K, which of the following value is the most close to the change of molar entropy for the solution with $x_A=0.3$?
(a) 16 J/K (b) 8 J/K (c) 0 kJ/K (d) -8 J/K (e) -16 J/K

64. Which of the following value is the most close to the molar entropy difference at $T= 300$ K between the solution with $x_A=0.3$ and that with $x_A=0.7$?
(a) 0 J/K (b) 1.4J/K (c) 2.4 J/K (d) 3.4 J/K (e) 4.4 J/K

Question 65-67

One mole of solid Fe at 1810 K is dissolved in a large volume of a liquid Henrian solution of Fe and Ni in which X_{Fe}=0.01, k_{Fe}=0.4, and which is also at 1810 K. Assume that heat of fusion and melting point of iron is ΔH_m= 13770 J/mole and T_m=1810 K respectively. Using the look-up table above and R=8 J/K-mole, answer the following questions.

65. Which of the following value is the most close to ΔG^M of forming Fe-Ni Henrian solution (X_{Fe}=0.01, T=1810 K)?
(a) -1 kJ (b) 0 kJ (c) 1 kJ (d) 2 kJ (e) 3 kJ

66. Which of the following value is the most close to the change in Gibbs free energy of solution caused by the addition?
(a) -80 kJ (b) 0 kJ (c) 40 kJ (d) 80 kJ (e) 160 kJ

67. Which of the following value is the most close to the change in enthalpy of solution caused by the addition?
(a) -14 kJ (b) -7 kJ (c) 0 kJ (d) 7 kJ (e) 14 kJ

Question 68-69

Melts in the system Al-Cu exhibit regular solution behavior. At 1000 K, a_{Al} = 0.1 in a liquid solution of x_{Al} = 0.01. Using the look-up table above and R=8 J/K-mole, answer the following questions.

68. Which of the followings is the most close to the value of Ω ?
(a) -18.8 kJ (b) -9.4 kJ (c) -4.7 kJ (d) 0 kJ (e) 18.8 kJ

69. Which of the following temperature is the most close to T_{cr} ?

(a) 1375 K (b) 1275 K (c) 1175 K (d) 1075 K (e) 975 K

Question 70-72

The binary system formed with component A and B exhibits complete ranges of liquid and solid ideal solution. The standard molar Gibbs free energy of melting for each component is

$\Delta G^o_{m,A} = 45 - 0.03T$

$\Delta G^o_{m.B} = 30 - 0.03T$

wherein the free energy is in unit of kJ and temperature in Kelvin degree. Using the look-up table above and R=8 J/K-mole, answer the following questions.

70. $T_{m,\,(A)}$ is

(a) 900 K (b) 1100 K (c) 1300 K (d) 1500 K (e) 1700 K

71. Which of the followings is the most close to the temperature at which $a_{B(s)}/a_{B(l)} = 0.5$?

(a) 1030 K (b) 1230 K (c) 1430 K (d) 1630 K (e) 1830 K

72. Around 1160 K, one end of the tie line shows $x_{A(s)}$ =0.48. Which of the followings is the most close to the value of $x_{A(l)}$ at the other end of the tie line?

(a) 0.16 (b) 0.48 (c) 0.64 (d) 0.84 (e) 0.96

Question 73-78

A binary system formed with Cs and Rb shows a complete range of solid and liquid solution, and also a congruent melting occurred at T_{cong} = 283 K and x_{Rb} =0.47.

For Cs: $\Delta G^o_{m,Cs} = 2100 - 7T$ (J)

Rb: $\Delta G^o_{m,Rb} = 2200 - 7T$ (J) .

Assume that the liquid solution is ideal and that the solid solution is regular. Using the look-up table above and R=8 J/K-mole, answer the following questions.

73. Which of the followings is correct?

(a) $T_{m,Cs} > T_{m,Rb} > T_{cong}$

(b) $T_{m,Rb} > T_{m,Cs} > T_{cong}$

(c) $T_{m,Rb} > T_{cong} > T_{m,Cs}$

(d) $T_{cong} > T_{m,Cs} > T_{m,Rb}$

(e) $T_{m,Cs} > T_{cong} > T_{m,Rb}$

74. Which of the followings is the most close to the value of $\Delta G^o{}_{m,Cs}$ at 283 K?

(a) -119 J (b) -99 J (c) 99 J (d) 119 J (e) 139 J

75. Which of the followings is the most close to the value of $\Delta G^o{}_{m,Rb}$ at 283 K?

(a) -239 J (b) -139 J (c) 139J (d) 219 J (e) 670 J

76. Relative to the pure solids as standard states at the temperature T, the expression for ΔG^M of liquid solution is

(a) $X_{Rb}\Delta G^o_{m,Rb} - X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs})$

(b) $X_{Rb}\Delta G^o_{m,Rb} + X_{Cs}\Delta G^o_{m,Cs} - RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs})$

(c) $RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs})$

(d) $-X_{Rb}\Delta G^o_{m,Rb} - X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs})$

(e) $X_{Rb}\Delta G^o_{m,Rb} + X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs})$

77. The expression for ΔG^M of solid solution is

(a) $X_{Rb}\Delta G^o_{m,Rb} - X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs}) + \Omega X_{Rb}X_{Cs}$

(b) $X_{Rb}\Delta G^o_{m,Rb} + X_{Cs}\Delta G^o_{m,Cs} - RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs}) + \Omega X_{Rb}X_{Cs}$

(c) $RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs}) + \Omega X_{Rb}X_{Cs}$

(d) $-X_{Rb}\Delta G^o_{m,Rb} - X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs}) + \Omega X_{Rb}X_{Cs}$

(e) $X_{Rb}\Delta G^o_{m,Rb} + X_{Cs}\Delta G^o_{m,Cs} + RT(X_{Rb}\ln X_{Rb} + X_{Cs}\ln X_{Cs}) + \Omega X_{Rb}X_{Cs}$

78.Which of the following value is the most close to that of Ω?

(a) -670 J (b) -570 J (c) -370J (d) 0 kJ (e) 670 J

79.Consider the decomposition of nitrogen gas, $N_2 = 2N$. At 3000 K, the equilibrium constant for the decomposition K_P is 3.5×10^{-11}, and both N_2 and N behave like an ideal gas. Which of the following is the correct description of the decomposition?

(a) K_P is constant with P.

(b) K_P is constant with T.

(c) K_P decreases with P.

(d) K_P increases with P.

(e) K_P decreases with T.

80.the standard Gibbs free energy of formation of NiO from $Ni_{(l)}$ is

$2Ni_{(l)} + O_2 = NiO$; $\Delta G^o = -506180 + 192\,T$ Assuming that the look-up table above and R=8 J/K-mole, which of the followings is the most close to the temperature for pure NiO to decompose in $P_{O2} = 10^{-3}$ atm?

(a) 1850 K (b) 2050 K (c) 2250 K (d) 2450 K (e) 2650 K