

94 學年度 \_\_\_\_\_ 材料系 \_\_\_\_\_ 系 (所) \_\_\_\_\_ 組碩士班入學考試

科目 \_\_\_\_\_ 理工測驗二 \_\_\_\_\_ 科目代碼 1302 共 13 頁第 1 頁 \*請在試卷【答案卷】內作答

1. Which of the following equations is INCORRECT?

(a)  $x = x(u, v), y = y(u, v)$ , then  $\frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial y} = 1$

(b)  $x = x(u, v), y = y(u, v)$ , then  $\frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial x} = 0$

(c)  $f(x, y, u, v) = 0$  and  $x = x(u, v), y = y(u, v)$ , then  $\frac{\partial f}{\partial u} + \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u} = 0$

(d)  $x = r \cos \theta, y = r \sin \theta$ , then  $\frac{\partial r}{\partial x} = \cos \theta$

(e)  $\frac{d}{dt} \int_{a(t)}^{b(t)} f(x, t) dx = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t} f(x, t) dx + b(t) f(b(t), t) - a(t) f(a(t), t)$

2. The tangent plane at the point  $(1, 3, -2)$  on the surface  $3x^2 + y^2 + z^2 = 16$  is

(a)  $3x + y + z = 4$                       (b)  $3x + 3y - 2z = 16$                       (c)  $x + 3y - 2z = 14$

(d)  $3x + 3y + 2z = 8$                       (e)  $3x + y - z = 8$

3. The spherical coordinates  $(\rho, \phi, \theta)$  of a point  $P$  are defined by that  $\rho$  is the distance from origin  $O$  to  $P$ ,  $\phi$  the angle between the  $z$  axis and  $OP$ ,  $\theta$  the angle between the  $x$  axis and the projection of  $OP$  on the  $x$ - $y$  plane. Which of the following relationships between the Cartesian coordinates  $(x, y, z)$  and the spherical coordinates  $(\rho, \phi, \theta)$  is correct?

(a)  $x = \rho \cos \phi \cos \theta$                       (b)  $y = \rho \sin \phi \cos \theta$                       (c)  $x = \rho \cos \phi \sin \theta$

(d)  $y = \rho \sin \phi \sin \theta$                       (e) none of the above

4. Following the previous problem, which of the equations for the unit vectors  $\hat{\rho}, \hat{\phi}, \hat{\theta}$  of the spherical coordinates is correct?

(a)  $\hat{\rho} = \cos \phi \cos \theta \hat{i} + \cos \phi \sin \theta \hat{j} + \sin \phi \hat{k}$                       (b)  $\hat{\rho} = \sin \phi \sin \theta \hat{i} + \sin \phi \cos \theta \hat{j} + \cos \phi \hat{k}$

(c)  $\hat{\phi} = \sin \phi \cos \theta \hat{i} + \sin \phi \sin \theta \hat{j} + \cos \phi \hat{k}$                       (d)  $\hat{\phi} = \sin \phi \cos \theta \hat{i} + \cos \phi \sin \theta \hat{j} - \cos \phi \hat{k}$

(e)  $\hat{\theta} = \cos \theta \hat{i} + \sin \theta \hat{j}$

5. Following the previous problem, the differential vector  $d\mathbf{R}$ , where  $\mathbf{R}$  is the position vector, is equal to

- (a)  $d\rho\hat{\rho} + d\phi\hat{\phi} + d\theta\hat{\theta}$       (b)  $\rho d\rho\hat{\rho} + \rho d\phi\hat{\phi} + \theta d\theta\hat{\theta}$       (c)  $d\rho\hat{\rho} + \rho d\phi\hat{\phi} + \rho \sin\phi d\theta\hat{\theta}$   
 (d)  $\rho d\rho\hat{\rho} + \rho \sin\phi d\phi\hat{\phi} + d\theta\hat{\theta}$       (e) none of the above

6. The integral  $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx$  is equal to

- (a) 1/2      (b) 1      (c) 2      (d)  $\pi$       (e)  $\pi/2$

7. The volume of the region enclosed by the four planes  $x = 0$ ,  $y = 0$ ,  $z = 0$ , and  $3x + 2y + 6z = 6$  is equal to

- (a)  $\int_0^2 \int_0^{3-\frac{3}{2}x} \int_0^{1-\frac{x}{2}-\frac{y}{3}} dz dy dx$       (b)  $\int_0^2 \int_0^{6-3x} \int_0^{1-\frac{x}{2}-\frac{y}{3}} dz dy dx$   
 (c)  $\int_0^2 \int_0^{3-\frac{3}{2}x} \int_0^{6-3x-2y} dz dy dx$       (d)  $\int_0^2 \int_0^{6-3x} \int_0^{6-3x-2y} dz dy dx$       (e) none of the above

8. A vector field  $\mathbf{V}(x, y, z) = xyz\hat{\mathbf{i}} - 2y^2\hat{\mathbf{k}}$ , then  $\text{curl } \mathbf{V}$  is equal to

- (a)  $-4y\hat{\mathbf{i}} + xy\hat{\mathbf{j}} + xz\hat{\mathbf{k}}$       (b)  $4y\hat{\mathbf{i}} + xy\hat{\mathbf{j}} - xz\hat{\mathbf{k}}$       (c)  $-4y\hat{\mathbf{i}} + xy\hat{\mathbf{j}} - xz\hat{\mathbf{k}}$   
 (d)  $4y\hat{\mathbf{i}} - xy\hat{\mathbf{j}} + xz\hat{\mathbf{k}}$       (e)  $-4y\hat{\mathbf{i}} - xy\hat{\mathbf{j}} + xz\hat{\mathbf{k}}$

9. If  $P$  and  $Q$  are scalar fields,  $\mathbf{A}, \mathbf{B}$  are vector fields,  $ds$  the line element vector,  $da$  the area element vector, and  $dv$  the volume element, which of the following equations is INCORRECT?

- (a)  $\nabla \times \nabla P = 0$       (b)  $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) + \nabla^2 \mathbf{A}$       (c)  $\int_{x_1}^{x_2} \nabla P \cdot ds = P(x_2) - P(x_1)$   
 (d)  $\int (\nabla \times \mathbf{A}) \cdot da = \oint \mathbf{A} \cdot ds$       (e)  $\int (P\nabla^2 Q - Q\nabla^2 P) dv = \oint (P\nabla Q - Q\nabla P) \cdot da$

10. Consider a path defined by the connection of two quadrant circles with a radius of 2. The first is on the  $x$ - $y$  plane from  $(2, 0, 0)$  to  $(0, 2, 0)$  and the second is on the  $y$ - $z$  plane from  $(0, 2, 0)$  to  $(0, 0, 2)$ . For a vector field

$\mathbf{V} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ , the line integral  $\int_C \mathbf{V} \cdot ds$  ( $ds$  is the line element vector) along the path is equal to

- (a) 1      (b) 2      (c) -1      (d) -2      (e) 0

11. Which of the following equations is correct?

- (a)  $\int_0^L \cos \frac{\pi x}{L} \cos \frac{2\pi x}{L} dx = \frac{L}{2}$       (b)  $\int_0^L \cos \frac{\pi x}{L} \cos \frac{3\pi x}{L} dx = \frac{L}{2}$   
 (c)  $\int_0^L \sin \frac{\pi x}{L} \sin \frac{2\pi x}{L} dx = \frac{L}{2}$       (d)  $\int_0^L \sin \frac{\pi x}{L} \sin \frac{3\pi x}{L} dx = \frac{L}{2}$       (e)  $\int_0^L \cos \frac{\pi x}{L} \cos \frac{\pi x}{L} dx = \frac{L}{2}$

12. The Fourier transform  $\hat{f}(\omega)$  of  $f(x)$  is defined by  $\hat{f}(\omega) = F\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$ . Which of the following relationships is correct?

- (a)  $f(x) = \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x} d\omega$       (b)  $F\{f(x-a)\} = e^{-ia\omega} \hat{f}(\omega)$       (c)  $F\{f(ax)\} = \frac{1}{a} \hat{f}(\omega)$   
 (d)  $F\{f(-x)\} = \hat{f}(\omega)$       (e)  $\hat{g}(\omega)$  is the Fourier transform of  $g(x)$ , then  $F\{f(x)g(x)\} = \hat{f}(\omega)\hat{g}(\omega)$

13. Following the previous problem, if  $H(x)$  is the Heaviside step function and  $a > 0$ ,  $F\{H(x) e^{-ax}\}$  is equal to

- (a)  $\frac{1}{a+i\omega}$       (b)  $\frac{1}{a-i\omega}$       (c)  $\frac{2}{a+i\omega}$       (d)  $\frac{2}{a-i\omega}$       (e) none of the above

14. For the eigenvalue problem  $y'' + \lambda y = 0$ , ( $0 < x < L$ ),  $y'(0) = y'(L) = 0$ , the eigenvalues are  $\lambda_n$  and the eigenfunctions are  $\phi_n$ . Which of the following is correct?

- (a) all  $\lambda_n > 0$       (b)  $\lambda_n = \frac{n^2 \pi^2}{L^2}$ ,  $n = 1, 2, 3, \dots$       (c) one of  $\lambda_n$  is 0      (d)  $\phi_n = \cos \frac{n\pi x}{L}$ ,  $n = 1, 2, 3, \dots$   
 (e) A function  $f(x)$  can be expanded as  $f(x) = \sum_n \left\{ \int_0^L f(x) \phi_n(x) dx \right\} \phi_n(x)$

15. Consider the diffusion equation  $\alpha^2 u_{xx} = u_t$ ,  $\frac{\partial u}{\partial x}(0, t) = u(L, t) = 0$ ,  $u(x, 0) = f(x)$  for  $0 \leq x \leq L$ ,  $0 \leq t$ . By

the method of separation of variables, we have  $u(x, t) = X(x)T(t)$  and  $\frac{X''}{X} = \frac{T'}{\alpha^2 T} = \text{constant} = -\kappa^2$ ,

$\kappa > 0$ . If  $n = 1, 2, 3, \dots$ ,  $\kappa$  is equal to

- (a)  $\frac{n\pi}{L}$       (b)  $\frac{n\pi}{2L}$       (c)  $\frac{2n\pi}{L}$       (d)  $\frac{(2n-1)\pi}{2L}$       (e)  $\frac{(2n-1)\pi}{L}$

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16. Following the above problem,  $u(x,t)$  is the superposition of the possible  $X(x)T(t)$  and has the form

$$u(x,t) = \sum_{n=1}^{\infty} A_n X_n(x) T_n(t). \text{ Then } X_n(x) \text{ is equal to}$$

- (a)  $\sin \frac{n\pi}{L} x$       (b)  $\cos \frac{n\pi}{2L} x$       (c)  $\sin \frac{2n\pi}{L} x$       (d)  $\cos \frac{(2n-1)\pi}{2L} x$       (e)  $\sin \frac{(2n-1)\pi}{L} x$

17.  $(1+i)^{1/3}$  is equal to (for  $k=0,1,2$ )

- (a)  $2^{1/6} e^{i(8k+1)\pi/12}$       (b)  $2^{1/6} e^{i(6k+1)\pi/12}$       (c)  $2^{1/6} e^{i(4k+1)\pi/12}$       (d)  $2^{1/6} e^{i(2k+1)\pi/12}$       (e)  $2^{1/6} e^{i(k+1)\pi/12}$

18. Let  $f(z) = M(x,y) + iN(x,y)$  be a complex function, and  $M$  and  $N$  are the real and imaginary parts. If  $f$  is differentiable,  $df/dz$  is equal to

- (a)  $\frac{\partial M}{\partial x} - i \frac{\partial M}{\partial y}$       (b)  $\frac{\partial M}{\partial x} - i \frac{\partial N}{\partial y}$       (c)  $\frac{\partial M}{\partial y} - i \frac{\partial M}{\partial x}$       (d)  $\frac{\partial N}{\partial x} - i \frac{\partial N}{\partial y}$       (e)  $\frac{\partial N}{\partial y} - i \frac{\partial N}{\partial x}$

19. On the complex  $z$  plane, let  $C$  be a closed circle centered at  $z=0$  with a radius of 2 and orientedcounterclockwise. The integral  $\oint_C \frac{e^z}{z^2 + 2z + 1} dz$  is equal to

- (a) 0      (b)  $\pi e i$       (c)  $\frac{\pi}{e} i$       (d)  $2\pi e i$       (e)  $\frac{2\pi}{e} i$

20. On the complex  $z$  plane, let  $C$  be a closed circle centered at  $z=0$  with a radius of 2 and orientedcounterclockwise. The integral  $\oint_C \sin \frac{1}{z} dz$  is equal to

- (a)  $\pi i$       (b)  $2\pi i$       (c)  $-\pi i$       (d)  $-2\pi i$       (e) 0

21. The general solution of  $3xy^2 y' + x^2 - 2y^3 = 0$  is

- (a)  $ye^{\frac{y^3}{x^2}} = C$       (b)  $ye^{\frac{y^2}{x^3}} = C$       (c)  $xe^{\frac{y^2}{x^3}} = C$       (d)  $xe^{\frac{y^3}{x^2}} = C$       (e) none of the above

22. The general solution of  $(x+y)y'+1=0$  is

- (a)  $\ln|x+y-1|+y=c$       (b)  $\ln|x-y-1|+y=c$       (c)  $\ln|x+y+1|+y=c$   
 (d)  $\ln|x-y+1|+y=c$       (e) none of the above

23. For metals, the permittivity (i.e.,  $\epsilon$ ) is negative and permeability (i.e.,  $\mu$ ) is positive in the range of microwave frequencies (i.e.,  $\omega$ ). As a result, the general solution of wave propagation equation  $E''(x)+\omega^2\epsilon\mu E(x)=0$  in microwave frequencies is

- (a)  $E(x)=Ae^{(\omega\sqrt{\epsilon\mu})x}+Be^{-(\omega\sqrt{\epsilon\mu})x}$       (b)  $Ae^{-(\omega\sqrt{\epsilon\mu})x}+Bxe^{-(\omega\sqrt{\epsilon\mu})x}$       (c)  $Ae^{(\omega\sqrt{\epsilon\mu})x}+Bxe^{(\omega\sqrt{\epsilon\mu})x}$   
 (d)  $Ae^{i(\omega\sqrt{\epsilon\mu})x}+Be^{-i(\omega\sqrt{\epsilon\mu})x}$       (e) none of the above

24. The general solution of  $y'''-2y'+4y=0$  is

- (a)  $c_1e^x+e^{-x}(c_2\cos x+c_3\sin x)$       (b)  $y=c_1e^x+e^{-2x}(c_2\cos x+c_3\sin x)$       (c)  $c_1e^{2x}+e^x(c_2\cos x+c_3\sin x)$   
 (d)  $c_1e^{-2x}+e^x(c_2\cos x+c_3\sin x)$       (e)  $c_1e^{-2x}+e^x(c_2\cos 2x+c_3\sin 2x)$

25. The general solution of  $y''+5y'+6y=e^{-2x}$  is

- (a)  $y=c_1e^{2x}+c_2e^{-3x}+xe^{2x}$       (b)  $y=c_1e^{-2x}+c_2e^{-3x}+xe^{-3x}$       (c)  $y=c_1e^{-2x}+c_2e^{3x}+xe^{-2x}$   
 (d)  $y=c_1e^{-2x}+c_2e^{-3x}+c_3xe^{-2x}$       (e)  $y=c_1e^{-2x}+c_2e^{-3x}+xe^{-2x}$

26. The general solution of  $y''-2y'+y=e^x+x$  is

- (a)  $y=(c_1+c_2x)e^x-\frac{x^2}{2}e^x+x+2$       (b)  $y=(c_1+c_2x)e^x+\frac{x^2}{2}e^x-x+2$   
 (c)  $y=(c_1+c_2x)e^x+\frac{x^2}{2}e^x+x+2$       (d)  $y=(c_1+c_2x)e^x+\frac{x^2}{2}e^x+x-2$       (e) none of the above

27. Find out the solution of the series  $\sum_{n=1}^{\infty} \frac{1}{n!(n+2)}$ : (a) 1      (b) 1/2      (c) 1/3      (d) 1/4      (e) 1/5

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28. The convergence radius of  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{9^n}$ : (a) 1 (b)  $\sqrt{3}$  (c) 3 (d) 9 (e)  $\sqrt[3]{3}$
29. Find the Laplace transform of  $\operatorname{erf}(\sqrt{t})$ : (a)  $\frac{1}{(s+1)\sqrt{s}}$  (b)  $\frac{2}{\sqrt{\pi}} \frac{1}{(s+1)\sqrt{s}}$  (c)  $\frac{1}{s\sqrt{(s+1)}}$   
 (d)  $\frac{2}{\sqrt{\pi}} \frac{1}{s\sqrt{(s+1)}}$  (e)  $\frac{1}{s\sqrt{(s-1)}}$
30. Find the inverse Laplace transform of  $\frac{s+1}{s^3+s^2-6s}$ : (a)  $\frac{1}{6} + \frac{3}{10}e^{2t} - \frac{2}{15}e^{-3t}$  (b)  $\frac{-1}{6} - \frac{3}{10}e^{2t} - \frac{2}{15}e^{-3t}$   
 (c)  $\frac{1}{6} + \frac{3}{10}e^{2t} + \frac{2}{15}e^{-3t}$  (d)  $\frac{-1}{6} + \frac{3}{10}e^{2t} + \frac{2}{15}e^{-3t}$  (e) none of the above
31. Use the Euler method to compute  $y_1, y_2,$  and  $y_3$  for the specified initial-value problem  $y' = -y$  using step size  $h=2$ . In the following numbers, which one is not the correct number for  $y_1, y_2,$  or  $y_3$   
 (a) 0.8 (b) 0.76 (c) 0.64 (d) 0.512 (e) none of the above
32. The Laplace transform of  $(1-e^{-t})/t$  is  
 (a)  $\ln \frac{s+1}{s}$  (b)  $\ln \frac{s-1}{s}$  (c)  $\ln \frac{s}{s+1}$  (d)  $\ln \frac{s}{s-1}$  (e) none of the above
33. Which one is not the singular point of the given system:  $x' = \sin y; y' = x + y$   
 (a) 0 (b)  $\pi/2$  (c)  $\pi$  (d)  $2\pi$  (e)  $4\pi$
34. Use Gauss elimination to determine the solution set of  $x_1 + x_2 - x_3 + 2x_4 = 1, 3x_1 + x_2 + x_3 - 5x_4 = 9, 2x_1 + x_2 - x_4 = 5,$  and  $x_1 - x_2 + 4x_3 = 8$ . The incorrect number for the solution set is  
 (a) -1 (b) 0 (c) 1 (d) 3 (e) 4
35. Expand the vector  $(1, 0, 0, 0)$  in terms of the orthogonal basis  $\{e_1, e_2, e_3, e_4\}$  of  $R^4$ , where  $e_1=(2, 0, -1, -5), e_2=(2, 0, -1, 1), e_3=(0, 1, 0, 0), e_4=(1, 0, 2, 0)$ .  
 (a)  $\{-\frac{1}{15}, \frac{1}{3}, 0, \frac{1}{5}\}$  (b)  $\{\frac{1}{15}, -\frac{1}{3}, 0, \frac{1}{5}\}$  (c)  $\{\frac{1}{15}, \frac{1}{3}, 0, -\frac{1}{5}\}$  (d)  $\{\frac{1}{15}, \frac{1}{3}, 0, \frac{1}{5}\}$  (e) none of the above

36. The norms of vectors  $\vec{A}$  and  $\vec{B}$  are 4 and 5 respectively and the angle between them is  $75^\circ$ . Therefore, the dot product of  $\vec{A}$  and  $\vec{B}$  is (a) 4.88 (b) 4.98 (c) 5.08 (d) 5.18 (e) 5.28

37. Evaluate the determinant of the matrix  $\begin{bmatrix} b^2 + c^2 & ab & ca \\ ab & c^2 + a^2 & bc \\ ca & bc & a^2 + b^2 \end{bmatrix}$ .

- (a)  $3abc$  (b)  $4a^2b^2c^2$  (c) 0 (d)  $a^2+b^2+c^2$  (e) none of the above

38. The inverse matrix of  $\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$  is (a)  $\begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{-3}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{4} & \frac{-1}{8} & \frac{9}{8} \end{bmatrix}$  (b)  $\begin{bmatrix} \frac{1}{4} & \frac{-3}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{-1}{4} & \frac{-1}{8} & \frac{9}{8} \end{bmatrix}$

- (c)  $\begin{bmatrix} \frac{1}{4} & \frac{-3}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{-1}{4} & \frac{-1}{8} & \frac{9}{8} \end{bmatrix}$  (d)  $\begin{bmatrix} \frac{1}{4} & \frac{-3}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{9}{8} \end{bmatrix}$  (e)  $\begin{bmatrix} \frac{1}{4} & \frac{-3}{8} & \frac{3}{8} \\ \frac{-1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{1}{4} & \frac{-1}{8} & \frac{9}{8} \end{bmatrix}$

39. Which one is not the eigenspace of the following matrix  $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ ?

- (a)  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  (b)  $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$  (c)  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  (d)  $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$  (e) none of the above

40. Diagonalize the given matrix  $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ . Which one is the corresponding diagonal matrix?

- (a)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  (b)  $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  (c)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$  (e) none of the above

41. When an ideal gas undergoes an adiabatic process, which one in the following is correct? ( $\gamma = C_p/C_v$ , R is the gas constant)
- (a)  $PV^\gamma = \text{constant}$       (b)  $TV^\gamma = \text{constant}$       (c)  $PV^R = \text{constant}$       (d)  $TV^R = \text{constant}$   
 (e) none of the above
42. When an ideal gas undergoes an isothermal process, which is the work done by the system?
- (a)  $RT \ln \frac{V_1}{V_2}$       (b)  $RT \ln \frac{V_2}{V_1}$       (c)  $RT \ln \frac{P_2}{P_1}$       (d)  $RT \ln \frac{P_1 V_2}{P_2 V_1}$   
 (e) none of the above
43. The entropy of a system contained in an adiabatic enclosure
- (a) decreases during a reversible process      (b) increases during a reversible process.  
 (c) decreases during an irreversible process      (d) increases during an irreversible process.  
 (e) none of the above.
44. In a closed system of constant internal energy and constant volume, the criterion for equilibrium is:
- (a) The Gibbs free energy has its minimum value      (b) The entropy has its maximum value  
 (c) The enthalpy has its minimum value      (d) The Helmholtz free energy has its minimum value  
 (e) None of the above.
45. In a closed system of constant temperature and constant pressure, the criterion for equilibrium is:
- (a) The Gibbs free energy has its minimum value      (b) The entropy has its maximum value.  
 (c) The enthalpy has its minimum value.      (d) The Helmholtz free energy has its minimum value  
 (e) None of the above.
46. If  $\Omega_{\text{conf}}$  is the number of spatial configuration of a system and  $\Omega_{\text{th}}$  is the number of arrangement of particles among the energy levels, the total entropy of the system is:
- (a)  $k(\ln \Omega_{\text{conf}} + \ln \Omega_{\text{th}})$       (b)  $k(\ln \Omega_{\text{conf}})(\ln \Omega_{\text{th}})$       (c)  $(k \ln \Omega_{\text{conf}})(k \ln \Omega_{\text{th}})$   
 (d)  $k \ln(\Omega_{\text{conf}} + \Omega_{\text{th}})$       (e) None of the above.
47. In a two-component (A and B) system, phase 1 and phase 2 coexist in equilibrium. Which description is correct? G is the molar Gibbs free energy.
- (a)  $G^{(1)} = G^{(2)}$       (b)  $G^{(1)}_A = G^{(1)}_B$       (c)  $G^{(1)}_A = G^{(2)}_A$   
 (d)  $G^{(1)}_A + G^{(2)}_A = G^{(1)}_B + G^{(2)}_B$       (e) None of the above.



48. Which relationship is incorrect? (A is the Helmholtz free energy)

- (a)  $\left(\frac{\partial U}{\partial S}\right)_{V,comp} = \left(\frac{\partial H}{\partial S}\right)_{P,comp}$       (b)  $\left(\frac{\partial U}{\partial V}\right)_{S,comp} = \left(\frac{\partial A}{\partial V}\right)_{T,comp}$   
 (c)  $\left(\frac{\partial H}{\partial P}\right)_{S,comp} = \left(\frac{\partial G}{\partial P}\right)_{T,comp}$       (d)  $\left(\frac{\partial A}{\partial T}\right)_{V,comp} = \left(\frac{\partial G}{\partial T}\right)_{P,comp}$   
 (e) None of the above.

49. Which relationship is correct?

- (a)  $\left(\frac{\partial T}{\partial V}\right)_{S,comp} = \left(\frac{\partial P}{\partial S}\right)_{V,comp}$       (b)  $\left(\frac{\partial T}{\partial P}\right)_{S,comp} = -\left(\frac{\partial V}{\partial S}\right)_{P,comp}$   
 (c)  $\left(\frac{\partial S}{\partial V}\right)_{T,comp} = \left(\frac{\partial P}{\partial T}\right)_{V,comp}$       (d)  $\left(\frac{\partial S}{\partial P}\right)_{T,comp} = \left(\frac{\partial V}{\partial T}\right)_{P,comp}$   
 (e) None of the above.

50. Gold and copper form complete ranges of solid solution at temperatures between 410°C and 889°C, and at  $T_1$  (which is within this range), the excess molar Gibbs free energy of formation of the solid solution is given by:  $G^{XS} = \Omega X_{Au}X_{Cu}$ .

The  $\ln a_{Cu}$  is given by: ( $a_{Cu}$  is the activity of Cu)

- (a)  $\frac{\Omega}{RT} X_{Au}^2 + \ln X_{Au}$       (b)  $\frac{\Omega}{RT} X_{Cu}^2 + \ln X_{Cu}$       (c)  $\frac{\Omega}{RT} X_{Au}^2 + \ln X_{Cu}$   
 (d)  $\frac{\Omega}{RT} X_{Cu}^2 + \ln X_{Au}$       (e) None of the above.

51. Following the above question, if  $\ln a_{Cu}(T_1) = A$ , then  $\ln a_{Cu}$  at  $T_2$  is given by:

- (a)  $(A + \ln X_{Cu}) \frac{T_1}{T_2}$       (b)  $(A + \ln X_{Cu}) \frac{T_2}{T_1}$       (c)  $(A \frac{T_1}{T_2} + \ln X_{Cu})$   
 (d)  $(A \frac{T_2}{T_1} + \ln X_{Cu})$       (e) None of the above.

52. A binary regular solution (A-B), the activity coefficient  $\gamma$  follows the relation:  $\ln \gamma_A = \alpha X_B^2$ . The Henry's law is followed as  $X_A \rightarrow 0$ . The coefficient of Henry's law  $k (= a_A/X_A)$  is equal to:

- (a)  $\alpha$       (b)  $\exp(\alpha)$       (c)  $\alpha/RT$       (d)  $\exp(\alpha/RT)$       (e) None of the above.

53. In a two-component system,  $X_A$  and  $X_B$  are molar fractions of A and B. The molar enthalpy of solution is given by  $\Delta H^M = \Omega X_A X_B$ , the partial molar enthalpy of solution of A is equal to:  
 (a)  $\Omega X_B$  (b)  $\Omega(X_B - X_A)$  (c)  $\Omega X_B^2$  (d)  $\Omega(X_B - X_A)^2$  (e) None of the above.
54. It is frequently found that the extensive thermodynamic properties of only one component of a binary solution are amenable to experimental measurement. Which of the following can be applied to calculate the properties of the other component?  
 (a) Raoult's law (b) Henry's law (c) Gibbs-Helmholtz equation  
 (d) Gibbs-Duhem equation (e) None of the above.
55. Which one is the van der Waals equation? (a and b are positive values.)  
 (a)  $(P + \frac{a}{V^2})(V - b) = RT$  (b)  $(P - \frac{a}{V^2})(V + b) = RT$  (c)  $(P + a)(V + b) = RT$   
 (d)  $(P - a)(V + b) = RT$  (e) None of the above.
56. The melting temperature of ice is  $0^\circ\text{C}$  at 1 atm. When the pressure is increased to 10 atm, the melting temperature of ice is:  
 (a)  $0^\circ\text{C}$  (b)  $<0^\circ\text{C}$  (c)  $>0^\circ\text{C}$  (d) Depending on other parameters (e) None of the above
57. When a three-component system is in equilibrium (pressure is fixed at 1 atm) with zero degree of freedom, there must coexist  
 (a) one phase (b) two phases (c) three phases (d) four phases (e) None of the above.
58. One mole of an ideal gas A (at 1 atm) and one mole of an ideal gas B (at 2 atm) are mixed to form an ideal gas solution of final pressure of 1 atm. The change of Gibbs free energy due to mixing is given by  
 (a)  $2 RT \ln(1/2)$  (b)  $(1/2) RT \ln 2$  (c)  $2 RT \ln(1/2) + RT \ln 2$  (d)  $(1/2) RT \ln 2 + RT \ln(1/2)$   
 (e) None of the above.
59. Which one of the following is correct (in the case of binary solution)?  
 (a)  $\because \Delta S^M = -R(X_A \ln X_A + X_B \ln X_B)$ ,  $\therefore$  the solution is an ideal solution.  
 (b)  $\because G^{XS} = b X_A X_B$ ,  $\therefore$  the solution is a regular solution.  
 (c)  $\because \Delta H^M = a X_A X_B$ ,  $\therefore$  the solution is a regular solution.  
 (d)  $\because \Delta G^M = -RT(X_A \ln X_A + X_B \ln X_B)$ ,  $\therefore$  the solution is an ideal solution.  
 (e) None of the above.
60. Which one of the following is correct in the case of ideal solution?  
 (a)  $\Delta G^M = 0$  (b)  $\Delta H^M = 0$  (c)  $\Delta S^M = 0$   
 (d)  $\Delta A^M = 0$  (A is the Helmholtz free energy) (e) None of the above.

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61. Consider that one mole of ideal gas expands from  $V_1=10$  liter to  $V_2=20$  liter, through three different reversible paths, (1) isobarically, (2) isothermally, and (3) adiabatically, then the work done would be,  
 (a)  $W_1>W_2>W_3$  (b)  $W_1>W_3>W_2$  (c)  $W_2>W_3>W_1$  (d)  $W_2>W_1>W_3$  (e)  $W_3>W_1>W_2$
62. For one mole of monatomic ideal gas, what is the value of constant pressure heat capacity ( $C_p$ )?  
 (a) 1.5R (b) 2R (c) 2.5R (d) 3R (e) 3.5R
63. If nitrogen gas,  $N_2$ , behaved like ideal gas, what would be the value of constant volume heat capacity ( $C_v$ )?  
 (a) 1.5R (b) 2R (c) 2.5R (d) 3R (e) 3.5R
64. Assume that ammonia gas,  $NH_3$ , behaved like ideal gas, what would be the value of internal energy?  
 (a) 2RT (b) 2.5RT (c) 3RT (d) 3.5RT (e) 4RT.
65. According to Dulong and Petit rule, the constant volume heat capacity of solid element at high temperatures will be (a) 1.5R (b) 2R (c) 2.5R (d) 3R (e) 3.5R.
66. Which of the following can be used as a criterion for thermodynamic equilibrium?  
 (a)  $\Delta S \geq 0$  (b)  $\Delta S_{U,V} \geq 0$  (c)  $\Delta S_{T,P} \geq 0$  (d)  $\Delta S_{P,V} \geq 0$  (e)  $\Delta S_{T,V} \geq 0$ .
67. For monatomic ideal gas, which is true?  
 (a)  $\left(\frac{\partial V}{\partial T}\right)_P > 0$  (b)  $\left(\frac{\partial V}{\partial P}\right)_T > 0$  (c)  $\left(\frac{\partial U}{\partial V}\right)_T > 0$  (d)  $\left(\frac{\partial H}{\partial V}\right)_T > 0$  (e)  $\left(\frac{\partial H}{\partial P}\right)_T > 0$
68. Which of the following is **NOT true** for chemical reactions between pure solids, when  $T \rightarrow 0$  K?  
 (a)  $\Delta C_p \rightarrow 0$  (b)  $\Delta S \rightarrow 0$  (c)  $\left(\frac{\partial \Delta G}{\partial T}\right)_P \rightarrow 0$  (d)  $\left(\frac{\partial \Delta H}{\partial T}\right)_P \rightarrow 0$  (e)  $\left(\frac{\partial \Delta S}{\partial T}\right)_P \rightarrow 0$
69. At room temperature the constant pressure heat capacity,  $C_p$ , of solid elements A and B are 50 J/K-g.atom and 80 J/K-g.atom, respectively. What might be the  $C_p$  of solid compound  $A_2B$ ?  
 (a) 60 J/K-g.atom (b) 65 J/K-g.atom (c) 130 J/K-g.atom  
 (d) 105 J/K-g.atom (e) 180 J/K-g.atom,
70. In a ternary system, what is the maximum number of phases that can coexist at equilibrium?  
 (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

71. Which of the following equation is correct in a multi-component solution?

- (a)  $\mu_i = \left( \frac{\partial G}{\partial n_i} \right)_{T,P,n_j..}$  (b)  $\mu_i = \left( \frac{\partial A}{\partial n_i} \right)_{T,V,n_j..}$  (c)  $\mu_i = \left( \frac{\partial H}{\partial n_i} \right)_{S,P,n_j..}$  (d)  $\mu_i = \left( \frac{\partial U}{\partial n_i} \right)_{S,V,n_j..}$  (e) All are true.

72. Consider the oxidation reaction of metallic element,  $M(s) + O_2(g) = MO_2(s)$ , and  $\Delta G^0 = A + BT$ ,

- Then (a)  $A > 0, B > 0$  (b)  $A > 0, B < 0$  (c)  $A < 0, B > 0$  (d)  $A < 0, B < 0$  (e)  $A < 0, B = 0$ .

73. In a binary solution, which statement is always true?

- (a) Both solute and solvent obey Raoult's law at zero temperature.  
 (b) Both solute and solvent obey Raoult's law at high temperatures.  
 (c) Solute and solvent obeys Raoult's and Henry's law, respectively.  
 (d) Solute and solvent obeys Henry's and Raoult's law, respectively.  
 (e) None is true.

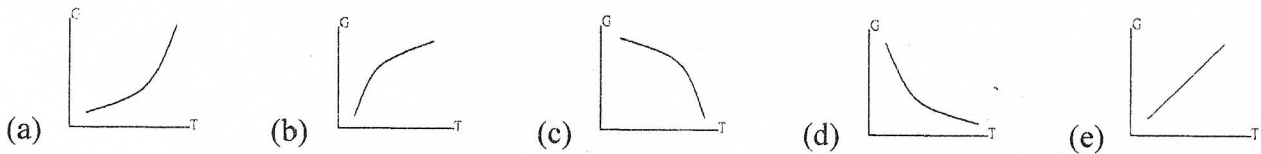
74. n moles of ideal gas A and (4-n) moles of an ideal gas B, each at 1 atm pressure, are mixed at constant total pressure and temperature. When the entropy is maximized, what is the value of n?

- (a) 1 (b) 2 (c) 2.5 (d) 3 (e) 4

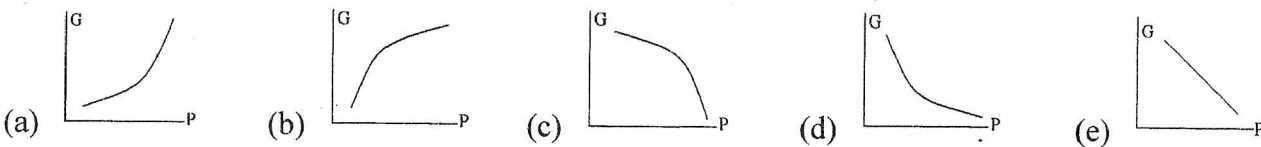
75. Two moles A and two moles B form an ideal solid solution, then the Gibbs free energy of mixing is,

- (a)  $+2RT \ln 2$  (b)  $+4RT \ln 2$  (c)  $-2RT \ln 2$  (d)  $-4RT \ln 2$  (e)  $-4RT \ln 4$

76. Which of the following curve represents the variation of Gibbs free energy with temperature for a pure single phase substance at a fixed pressure?



77. Which of the following curve represents the variation of Gibbs free energy with pressure for a pure single phase substance at a fixed temperature?



78. If a regular solution can be represented by  $G^{XS} = 1000RX_A X_B$  J/mole. What is the critical temperature,  $T_{cr}$ , for the decomposition of a homogeneous solution?

- (a) 125K (b) 250K (c) 398K (d) 500K (e) 523K

79. Consider the oxidation reaction of pure iron at room temperature, then

- (a)  $\Delta G^0 > 0$     (b)  $\Delta H^0 > 0$     (c)  $\Delta S^0 > 0$     (d)  $\Delta G^0$  increases as temperature increases  
(e)  $\Delta G^0$  decreases as temperature increases.

80. Consider the gas phase reaction,  $2A + 3B = C + 4D$ , and  $H_A^0 = 10$  J/mole,  $H_B^0 = 30$  J/mole,  $H_C^0 = 15$  J/mole,  $H_D^0 = 20$  J/mole, then

- (a) as temperature increases, equilibrium constant  $K_p$  increases  
(b) as temperature increases, equilibrium constant  $K_p$  decreases  
(c) as total pressure increases, equilibrium constant  $K_p$  increases  
(d) as total pressure increases, equilibrium constant  $K_p$  decreases  
(e) the effect of temperature or pressure on  $K_p$  cannot be determined