國立清華大學命題紙

八十八學年度 材料科學工程研究所(系) 系(所) 甲乙丙 組碩士班研究生招生考試

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注意：在前6題中，若某一題計算錯誤，該題分數可能給零分。

1. (5分)

   $f(t) = 3$ is a constant function. Use the integral-transform definition to compute its Laplace transform.

2. (5分)

   Use the Laplace transform to solve the initial value problem:

   $$\frac{dy}{dt} + 4y = 8 \qquad \text{with} \quad y(0) = 2.$$

   First, write down the Laplace transform $L\{\frac{dy}{dt}\}$.

3. (10分)

   Compute the Wronskian determinant of the two functions $u_1(x)$ and $u_2(x)$, where

   $$u_1(x) = x \quad \text{and} \quad u_2(x) = x^2.$$

4. (10分)

   Find a general solution $y(t)$ of the ordinary differential equation

   $$\frac{d^2y}{dt^2} + (6)\left(\frac{dy}{dt}\right) + (9)(y) = 0$$

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科目 工程數學 科號 1603 2603 1703 2703 1803 2803 1903 2903 共 3 頁第 2 頁 *請在試卷【答案卷】內作答

5. (10分)

Compute the nullity of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 3 & 6 & 3 & 0 & 0 \end{bmatrix};$$

that is, find the dimension of the null space of A, which has $m = 2$ rows and $n = 5$ columns. First, you must find the reduced row-echelon form of A to compute the nullity.

6. (10分)

Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Suppose that there is a modal matrix Q which diagonalizes A. This problem asks you to compute all the diagonal elements of the diagonal matrix $D = Q^{-1}AQ$. You can use theorems without proving them.

7. Find the steady –state temperature distribution (by Fourier series) in the semi –infinite region $0 \le x \le a$, $y \ge 0$ if the temperatures on the bottom and left sides are kept at zero, and the temperature on the right side is kept at constant T. Note that solutions are to be bounded. (10%)

8. Solve the wave equation by Fourier transform:

$$U_{tt} = c^2 U_{xx} \qquad (-\infty < x < \infty, t > 0)$$

subject to the conditions

$$U(x, 0) = f(x)$$
$$U_t(x, 0) = 0$$
$$U \Rightarrow 0, U_x \Rightarrow 0 \text{ as } |x| \Rightarrow \infty \text{ for all } t$$

where f (x) is assumed to have a Fourier transform. (10%)

※ $U_{tt}=\partial^2 U/\partial t^2$ $U_{xx}=\partial^2 U/\partial x^2$

$U_t=\partial U/\partial t$ $U_x=\partial U/\partial x$

9. Expand each of the following functions in a Laurent series that converges for $0<|z|< R$ and determines the precise region of convergence.

(a). e^z/z^2 (5%), (b). $1/z(1+z^2)$ (5%)

10. Evaluate the following integrals where C is the unit circle (counterclockwise)

(a). $\oint_C \cot(z/4)\, dz$ (5%), (b). $\oint_C \tan \pi z\, dz$ (5%).

11. Evaluate $\int_0^{\pi} [1/(\alpha + \beta \cos\theta)]\, d\theta$ where $\alpha > \beta > 0$ (10%)