

Any vector quantity is underlined in the following problems.

- Suppose that an infinitely long solenoid with radius  $R$ ,  $N$  turns per unit length, and current  $I$ , has a magnetic field  $\underline{B}$  pointing  $z$  direction. Coaxial with the solenoid are two long cylindrical shells of length  $L$ . One inside the solenoid at radius  $a$ , carries a charge  $-2Q$  uniformly distributed over its surface; the other, outside the solenoid at radius  $b$ , carries charge  $+2Q$ .  $L$  is supposed to be much larger than  $b$ .
  - Find out the electric field between two cylinders. (5 points)
  - Find out the divergence of the electric field at a certain point of the outer cylinder. (5 points)
  - Find out the total momentum stored in the system. (5 points)
  - Find out the energy flux out of the outer cylinder outside the solenoid. (5 points)
  - Find out Maxwell stress tensor  $T_{ij}$  on the surface of the outer cylinder, not including the surfaces at two ends. (5 points)
  - Find out the rotation direction of the outer cylinder when the current  $I$  in the solenoid is gradually decreased to zero? (5 points)
- Suppose that a free charge  $Q$  is initially put on the surface of a semiconducting Si(100) substrate of  $4 \text{ cm}^2$  in size. Please calculate the time required to dissipate one half of the free charge  $Q$ . You may assume any material's constant if the material's data is needed in the calculation. (10 points)
- The index of refraction of diamond is 2.42. Suppose that a "yellow" light ray is directed into an abrupt air/diamond interface at normal incidence. Assume that permeability of diamond is the same as that of air. Please calculate the speed of yellow light in diamond. (10 points)

國立清華大學 命題紙

八十七學年度 材料科學工程研究所(系) (所) 電二 組碩士班研究生入學考試

科目 電 磁 學 科號 1703 共 2 頁第 2 頁 \*請在試卷【答案卷】內作答

4. An electron is placed in vacuum at a distance  $x$  from a conducting plane.
- (a) What is the induced charge on the plane? What is the force on the electron due to the coulomb attraction with the induced charge? What is the potential energy of the electron? (5 points)
- (b) If an electric field  $E$  (in  $\bar{x}$  direction) is applied to the vacuum, the potential energy of the electron is modified. Please find the resulting potential energy of the electron and show that it has an extreme value of  $-e \sqrt{\frac{eE}{4\pi\epsilon_0}}$ . (8 points)
5. (a) Write down the Maxwell's equations for both electrostatics and magnetostatics. Also give names to each equation if applicable. (4 points)
- (b) What are the boundary conditions for the electric field vector at the interface of two dielectrics, when a surface charge of density  $\sigma$  is present at the interface? (3 points)
- (c) A long copper rod of radius  $R$  carries a steady current  $I$ , which is distributed in such a way that  $J$  is proportional to  $r$ , the distance from the axis. Find the magnetic field both inside and outside the rod. (5 points)
6. A long rectangular pipe, running parallel to the  $z$  axis, has three grounded metal sides, at  $y = 0$ ,  $y = \pi$ , and  $x = 0$ . The fourth side, at  $x = a$ , is maintained at a specified potential  $V_0(y)$ .
- (a) Develop a general formula for the potential within the pipe. (10 points)
- (b) Find the potential explicitly, for the case  $V_0(y) = V_0$  (a constant). (5 points)
7. A metal sphere of radius  $a$  carries a charge  $Q$ . It is surrounded, out to radius  $b$ , by linear dielectric material of permittivity  $\epsilon$ . Find the potential at the center (relative to infinity). (10 points)