科目 工程數學　類組別 031　

＊請在試卷答案卷（卡）內作答

1. (10%) Consider the vector function $\vec{F}(x,y,z)=(3x-y)\,\vec{i}+(y-4xz)\,\vec{j}-z\,\vec{k}$ and the surface S bounding the sphere of radius 4 about $(1,1,1)$. Evaluate $\iint_S \vec{F}\cdot\vec{n}dA$.

2. (15%) Solve the linear system: $\begin{cases} x_1 - x_2 + 3x_3 - x_4 = 3 \\ x_2 - 3x_3 + 2x_4 = -2 \\ x_1 + 2x_2 - x_4 = 9 \end{cases}$.

3. Consider the matrix $\mathbf{A}=\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$.

(1) (5%) Find the inverse of **A**.

(2) (10%) A rotation matrix of the form $\mathbf{R}=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ can be used to diagonalize **A** to a diagonal matrix **D** (*i.e.* $\mathbf{D}=\mathbf{R}'\mathbf{A}\mathbf{R}$). Determine **R** and **D**.

4. (15%) Consider the ODE: $(2y^2-9xy)dx+(3xy-6x^2)dy=0$. Given $y(1)=1$, solve the initial value problem.

5. (15%) Find the general solution of the ODE: $y'''-5y''+2y'+8y=200\sin 2x$.

6. (5%) Find the Laplace Transform of $f(t)=2t^2e^{-3t}+3\sin 6t$.

(A) $\dfrac{4e^{3s}}{s^3}+\dfrac{18}{s^2+6}$;　(B) $\dfrac{4}{(s+3)^3}+\dfrac{18}{s^2+36}$;

(C) $\dfrac{4}{(s+3)^2}+\dfrac{3s}{s^2+36}$;　(D) $\dfrac{12}{(s-3)^3}+\dfrac{3}{s^2-36}$.

7. Given $\mathcal{F}\left\{e^{-ax^2}\right\}=\dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-ax^2}e^{-i\omega x}dx=\dfrac{1}{\sqrt{2a}}e^{-\omega^2/4a}$ for $a>0$,

(1) (8%) Find the Fourier transform of $f(x)=e^{-(x-3)^2/4}$.

(2) (7%) Find the Fourier transform of $g(x)=(x-3)e^{-(x-3)^2/4}$.

8. (10%) Given Gamma function, $\Gamma(\nu)=\int_0^{\infty}e^{-t}t^{\nu-1}dt$ $(\nu>0)$, show that $\Gamma(n+1)=n!$ for integer n.