科目 工程數學 類組別 033

*請在試卷答案卷（卡）內作答

Engineering Mathematics

1. (5%) Reduce, by letting $u(x)=[y(x)]^{1-a}$ where a is a constant, to a linear form and solve the following differential equation: (Section 1.6, Prob. 33)

$$y'+\frac{1}{3}y=\frac{1}{3}(1-2x)y^4$$

2. (5%) Converting the given equation $y''-9y=0$ to a system $\vec{y}'=\vec{A}\vec{y}$, where $\vec{A}$ is a 2×2 matrix, and then determining $\vec{A}$. (section 3.1, Prob. 9)

3. (10%) Given the differential equation (Equation (2) on Page 108)

$$y''+p(x)y'+q(x)y=r(x) \qquad (1)$$

with arbitrary variable function $p(x)$, $q(x)$, and $r(x)$ that are continuous on some interval I. Use the method of variation to show a particular solution of (1) on I in the form

$$y_p(x)=-y_1\int\frac{y_2 r}{W}dx+y_2\int\frac{y_1 r}{W}dx$$

where y_1, y_2 form a basis of solutions of the homogeneous equation corresponding to (1), and

$$W=-y_1 y_2'-y_2 y_1'$$

4. (10%) Find a power series solution in powers of x of the following differential equation $y'=-2xy$. (Section 4.2 prob. 1)

5. (10%) Solve the system of equation (section 6.6 Prob. 19)

$$\begin{aligned} 3x+7y+8z&=-13\\ 2x\qquad+9z&=-5\\ -4x+y-26z&=2 \end{aligned}$$

by Gramer's rule.

6. (10%) Given $\vec{A}=\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$. Find the eigenvalues of $\vec{A}$, and also diagonalization of $\vec{A}$. (section 6.7 prob. 1)

7. (10%) A vector field is given by $\vec{u}=y^2\vec{i}+2xy\vec{j}-z^2\vec{k}$. Determine the

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divergence of $\vec{u}$ and curl of $\vec{u}$ at the point $(1,2,1)$. Also, determine if the vector field is solenoidal or irrotational.

8. (10%) Use Stroke's theorem to evaluate $\oint_C z^2 \exp(x^2)dx + xy^2dy + \tan^{-1} ydz$, where C is the circle $x^2 + y^2 = 1$.

9. (a) (5%) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

   (b) (5%) By (a), find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

10. (10%) Find the Fourier series of

$$f(x) = \begin{cases} 1 & ,-1 < x < 1 \\ x & ,0 \le x \le 1 \end{cases}$$

11. (10%) Solve $y\frac{\partial^2 u}{\partial x \partial y} + u = 0$.