

科目 工程數學-微分方程 類組別 C1 共 1 頁第 1 頁 *請在試卷答案卷內作答

1. Solve the following differential equations. (40%)

(a) $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$

(b) $xy' = 2y + x^3e^x$

(c) $y'' + 2y' + y = xe^{-x}$

(d) $4y'' + \pi^2y = 0, \quad y(0) = 2, y'(0) = 0$

2. Consider the second-order homogeneous linear differential equation,

$$y'' + p(x)y' + q(x)y = 0 \dots\dots\dots(*)$$

(a) Let y_1 and y_2 be two solutions of Eq (*), prove that $c_1y_1 + c_2y_2$ is also a solution of Eq (*). (4%)

(b) Suppose $y_1 = \sin^2 x$ and $y_2 = \cos^2 x$, can they form a basis of Eq (*)? Why? (3%)

(c) Now consider the second-order non-homogeneous linear differential equation

$$y'' + p(x)y' + q(x)y = r(x), r(x) \neq 0 \dots\dots\dots(**)$$

and let y_1 and y_2 be two solutions of Eq (**), is $c_1y_1 + c_2y_2$ also a solution of Eq (*)? (3%)

3. Solve the following system of differential equations (15%)

$$y_1' = -y_1 + y_2, \quad y_2' = -y_1 - y_2; \quad y_1(0) = 0, \quad y_2(0) = 1$$

4. Solve the initial value problem (15%)

$$y'' + 3y' + 2y = f(t); \quad y(0) = 0, \quad y'(0) = 1$$

$$\text{in which } f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

5. Determine the Laplace transform of

(a) $f(t) = 1$. (5%)

(b) $f(t) = \frac{e^{-2t} \sin t}{t}$ (5%)

6. Determine the Laplace inverse transform of

(a) $F(s) = 1$. (5%)

(b) $\frac{e^{-5s}}{s(s+1)^2}$ (5%)