科目 微積分 類組別 A2, A3, A4, A5, A6, B6, B7 共 2 頁第 1 頁 *請在試卷答案卷（卡）內作答

一. 填充題： 請將答案**按字母順序寫在答案紙前八行。 不要寫計算過程。** (每格 8 分)

1. $\int_0^{\pi} x\sin x\,dx =$ __A__ .

2. For the equation $y^3 - xy^2 + \cos(xy) = 2$, $\dfrac{dy}{dx}$ at the point $(0,1)$ is __B__ .

3. Let $F(t) = \int_{g(t)}^{h(t)} f(u)\,du$, where f is continuous and g and h are differentiable. Find $F'(t) =$ __C__ .

4. The maximum value of $f(x,y) = y^2 - x^2$ on the ellipse $\dfrac{x^2}{4} + y^2 = 1$ is __D__ .

5. $\int_0^1 \int_x^1 e^{y^2}\,dy\,dx =$ __E__ .

6. $\lim_{x\to 0}\left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right) =$ __F__ .

7. Let $f:[0,3]\to[0,3]$ be a continuous function with $f(0)=0$ and $f(3)=3$. If f is one-to-one and $\int_0^3 f(x)\,dx = \dfrac{9}{5}$, then $\int_0^3 f^{-1}(x)\,dx =$ __G__ , where f^{-1} is the inverse function of f .

8. The radius of convergence of the power series $\sum_{k=1}^{\infty}\left(\dfrac{k+1}{k}\right)^{k^2} x^k$ is __H__ .

台灣聯合大學系統94學年度學士班轉學生考試命題紙

科目 微積分 類組別 A2、A3、A4、A5、A6、B6、B7 共 2 頁第 2 頁 *請在試卷答案卷（卡）內作答

二. 計算與證明：請詳細寫出每一個推導步驟。 (每題 12 分)

1. Let $f:\mathbb{R}\to\mathbb{R}$ be twice differentiable. If f'' is nowhere vanishing, then f has at most two distinct real roots.

2. Prove that $\int_0^1 e^{\frac{-t^2}{2}}\,dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k k!}$.

3. Evaluate $\oint_C (x^3+y^3)dx+(2y^3-x^3)dy$, where C is the unit circle.