

第一部分：多重選擇題，共四題，請在答案卷上註明題號(子、丑、寅、卯)後寫下答案。每題 4.5 分；各選項獨立計分，每答對一選項得 1.5 分，答錯倒扣 1 分，若整題完全不答則得零分。

1. If  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  are differentiable, which of the following statements are true? Ans: 子

- (a) If  $f(x) \geq g(x)$  for all  $x \in \mathbb{R}$ , then  $f'(x) \geq g'(x)$  for all  $x \in \mathbb{R}$ .
- (b) If  $f'(x) \geq g'(x)$  for all  $x \in \mathbb{R}$ , then  $f(x) \geq g(x)$  for all  $x \in \mathbb{R}$ .
- (c) If  $f'(x) \geq g'(x)$  for all  $x \in \mathbb{R}$ , then  $f(1) - g(1) \geq f(0) - g(0)$ .

2. Which of the following statements are true? Ans: 丑

- (a) If  $f: [0, 1] \rightarrow [0, 1]$  is a continuous function, then there exists a point  $c \in [0, 1]$  such that  $f(c) = c^2$ .
- (b)  $|\cos^2 x - \cos^2 y| \leq |x - y|$  holds for all  $x, y \in \mathbb{R}$ .
- (c) If  $f: [0, 1] \rightarrow \mathbb{R}$  is a continuous function such that  $\int_0^1 f(x) dx = 0$ , then  $f(x) = 0$  for some  $x \in [0, 1]$ .

3. Which of the following statements are true? Ans: 寅

- (a) If the series  $\sum_{n=1}^{\infty} a_n$  converges, then we have  $\lim_{n \rightarrow \infty} a_n = 0$ .
- (b) If the series  $\sum_{n=1}^{\infty} a_n$  diverges, then we have  $\lim_{n \rightarrow \infty} a_n \neq 0$ .
- (c) If the series  $\sum_{n=1}^{\infty} a_n$  converges, then the series  $\sum_{n=1}^{\infty} a_n^2$  also converges.

4. Let  $f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$ , which of the following statements are true? Ans: 卯

- (a)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exists.
- (b)  $f$  is continuous at  $(0, 0)$ .
- (c)  $\frac{\partial f}{\partial x}(0, 0)$  and  $\frac{\partial f}{\partial y}(0, 0)$  both exist.

第二部分：填充題，共七題，請在答案卷上註明題號(甲、乙、丙……)後寫下答案；每題 8 分；不需演算過程。

1. Suppose that  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a continuous function and that  $f(x) = \frac{\sin(1+x) - \sin(1-x)}{x}$  for  $x \neq 0$ . Then  $f(0) =$  甲

2. If  $y = y(x)$  satisfies the differential equation  $xy'' + y' = 9x^2$  for  $x > 0$ , and the conditions  $y(1) = y'(1) = 0$ , then  $y(x) =$  乙

3. The largest value of  $f(x) = x \cos x - \sin x$  on the interval  $[0, 2\pi]$  is 丙

4. Let  $a, b, c, d$  be real numbers. Then the maximum value of  $ac + bd$  subject to the constraints  $a^2 + b^2 = 1$  and  $(c - 6)^2 + (d - 4)^2 = 4$  is 丁

九十一學年度 計財系轉學生招生考試

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5. Suppose that when the production of a particular commodity requires  $x$  machine-hours and  $y$  person-hours, the cost of production is given by  $f(x, y) = 2x^3 - 6xy + y^2 + 4y + 1000$ . Determine the number  $m$  of machine-hours and the number  $p$  of person-hours needed to produce the commodity at the least cost. Ans:  $(m, p) =$   
戊

6.  $\int_{-1}^1 \int_{|y|}^1 (x+y)^2 dx dy =$  己

7. Suppose that  $f: [-1, 1] \rightarrow \mathbb{R}$  is a continuous function, then  $\lim_{h \rightarrow 0^+} \int_{-h}^h \frac{h}{h^2+x^2} f(x) dx =$   
庚

第三部分：計算證明題，共三題，請在答案卷上註明題號並寫下詳細過程。

1.(9%) (a) Show that  $\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$ .

(b) Find  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(5x^2+4xy+4y^2)} dx dy$ .

2.(9%) Consider the plane curve  $C: x^3 - xy + y^3 + 3x = 1$ .

(a) Prove that  $C$  intersects the  $x$ -axis at exactly one point, say  $(a, 0)$ .

(b) Find the equation of the line tangent to  $C$  at  $(a, 0)$ .

3.(8%) Find a differentiable function  $y = f(x)$  such that  $f(0) = 0$  and the arc length of the graph of  $f$  between  $(0, 0)$  and  $(x, y)$  is  $e^x + y - 1$ .